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Volume XLIII Number 3

Edited by William David Reeve

A Program for Determining the Mathematical Needs of Engineering Students*

By JAMES H. ZANT

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INTRODUCTION

THE curriculum of our high schools, including the part dealing with mathematics, I might say, especially the part dealing with mathematics, is largely traditional. It is true that there have been new courses introduced in some places and old courses have been reorganized in some schools but one who attended a high school 25 or 35 years ago would have little difficulty recognizing the material being taught if he were to go back to the classroom. This is especially true in mathematics. Without having statistical data on the subject we would be safe in saying that the majority of high schools, small and large, teach about the same algebra, geometry, etc. as has been taught for many years. In fact, it is not difficult to recognize the geometry of today in a copy of Playfair's *Euclid* written in 1813 and still in use in some schools in this country less than 75 years ago. This is not necessarily bad. If students need the mathematics that was taught 75 years ago or 2,000 years ago, as in the case of Euclid's geometry, there is every reason that such subject matter should continue to be included in our school curriculum. The important thing is the pupils' needs now and in the future. We should find what those needs are for all citizens and for special groups who are segregated for special training.

1. *Meaning of Mathematical Needs.* Since this paper deals with a particular group of students, that is, prospective engineers, the mathematical needs in question may be more easily defined and determined. We shall think of the mathematical needs of an engineering student as those skills, concepts and understandings which he will need to study the various phases of engineering and to practice it when he has completed the course. In a sense this ignores an important area of the prospective engineer's needs, namely, his general education for living in society as an intelligent citizen. Mathematical needs for citizenship should ideally be completed in the secondary school and since this phase of a student's education has been discussed in a previous paper, it will not be taken up here.[1] Hence the mathemati-

* Read at the summer meeting of the National Council of Teachers of Mathematics at Denver, Colo., August 29—Sept. 1, 1949.

[1] See Zant, James H. "What are the Mathematical Needs of the High School Student?" THE MATHEMATICS TEACHER, Vol. XLII, No. 2 (Feb., 1949) pp. 75–78. Also "The Second Report of the Commission on Postwar Plans," THE MATHEMATICS TEACHER, Vol. XXXIX, No. 5, pp. 195–221.

cal needs of the engineering student will be thought of as the needs of a specialized group. It will, therefore, not be necessary to justify the needs which will eventually be determined except to show that they are necessary for success either while the student is in college or when he is on the job after having finished his professional training.

2. *Specialized Mathematical Needs of Engineering Students* may be classified under at least three heads: (1) that mathematical knowledge which he needs to study the mathematics necessary for an engineering education, (2) that which he needs to study other courses in the engineering curriculum and (3) that which he needs to carry on his professional work after he has graduated. There is probably a considerable amount of agreement among college mathematics teachers and among engineering educators on what mathematics an engineering student needs. However, it is opinion, though based on long experience. The experience of most of us has been in teaching a fairly static curriculum and hence our opinions are colored by this experience. Even such opinion as that may not conform to the facts. For example, college mathematics courses in analytical geometry have traditionally included a considerable amount of work on polar coordinates on the assumption that this knowledge is vital to the study of the calculus. Most of us here have lamented the fact that we could not teach more, yet a popular textbook in calculus uses the polar equation of the circle only nine times and only three curves expressed in polar coordinates are used more than five times. One wonders if the writers of the textbooks in analytical geometry did not assume that this topic is more important than it actually is. The problem then is to find just what mathematics this particular group of students needs on the basis of the three categories suggested at the beginning of this paragraph.

I submit that such information is not available in a reliable form. If we are going to build a functional curriculum at the college level for this group of students, we must know exactly, if possible, what mathematics they will need now and in the future. Some work has been done in this field but it is all too little. Moreover, it should be a never-ending process of continually bringing the data up-to-date. Only in this way can we be sure that we are not wasting the students' time by teaching non-essentials and failing to give them some of the things badly needed in their work now or in the future. Hence the purpose of this paper is to suggest a program for doing this sort of thing and to call attention to some of the data which has been collected.

3. *The Suggested Procedure.* At the Oklahoma Agricultural and Mechanical College a research project sponsored jointly by the Department of Mathematics and the Research Foundation is designed to find specifically what skills, knowledge and understandings in algebra and trigonometry a person will need as an engineer. We are limiting it to this small area in order to be specific and to make it possible to complete the study in a reasonable length of time. Later we hope to extend it to the whole field of engineering mathematics. The method of procedure would need little modification to investigate other phases of the field.

It seemed more pertinent to begin this study with a required engineering course which unquestionably uses mathematics throughout. The course in *Statics* was chosen. In this college it is called *Civil Engineering 213, Mechanics, Statics.* It has a prerequisite of one semester of calculus, is required of all engineering students and consists of "the principles of applied mechanics as developed in statics, center of gravity and moment of inertia." The textbook in use is *Engineering Mechanics*, by Frank L. Brown. Courses of this sort are outside the experience of most mathematics teachers and the mathematics used, or rather some of the mathematics

not used, will probably come as a surprise to many on the mathematics staff. An incomplete examination of this book reveals that the larger part of the mathematics involved is algebra and trigonometry. However, both the algebraic and trigonometric skills involved are relatively simple. Much use is made of substituting in formulas, solving for one unknown, extracting roots, etc., but little of such topics as progressions, solving equations of higher degree, mathematical induction and the like. Much use is made of solving a right triangle for a side or an angle, the laws of sines and cosines, solution of problems where the angles are 30°, 60° and 45°, inverse functions, etc., but little of such topics as the law of tangents, two angle formulas, multiple angles, etc. These facts may be the reason for the statement of some engineering teachers that the trigonometry needed by an engineer can be taught in ten *easy* lessons. This will be discussed more fully later.

Since this course is made up entirely of textual material and solutions of certain problems from the book, we feel that a careful analysis of the mathematical skills and knowledge needed to read the text and solve the problems make up the mathematical needs of these students as far as the course in *Statics* is concerned. These will not be hard to find since it is merely a matter of solving the problems and counting the different operations used. When such a tabulation is made for all of the courses the student takes as a prospective engineer, it will be merely a matter of combining, eliminating duplicates and the like.

The problem of finding the needs of an engineer after he has graduated will be more difficult, since the work done by engineers is so varied. The mathematical skills will also vary because of the training of the persons doing the jobs. However, it should be possible to find something fairly definite along these lines. There are engineering reports and work sheets which should reveal the mathematics used on a particular job. There are papers written for publication by engineers in which they describe methods by which they have arrived at the solutions of their problems. A critical examination of these should reveal the needs of the persons preparing them and, if a typical selection is made, we should be able to say with some definiteness what the prospective engineer should be taught.

Before deciding just what should be included in a course in college algebra and trigonometry, we should also examine critically the mathematics needed in the more advanced courses in the subject. Since we have had long experience in this field we are more likely to accept the traditional subject matter in this area than we are in *Statics*, *Physics*, *Mechanics*, etc. The illustration given regarding polar coordinates indicates that we should be careful here also. Hence we must examine the advanced subject matter in mathematics just as critically as we have in the other courses. We must also consider the fact that our analysis of other activities of prospective engineers and engineers may make it necessary to increase the amount of mathematics offered. This has already occurred in many engineering curricula in which differential equations is now the last required subject where it was formerly calculus.

The need for calculus and other topics beyond that field make it impossible to cut the more elementary courses to the bare essentials needed in engineering courses and practice, that is, the ten *easy* lessons. For example, a knowledge of trigonometry far beyond its practical uses is necessary to study and understand analytics and calculus. Hence we must teach an engineering student what is called analytical trigonometry in addition to the practical formulas about sides and angles of triangles. In fact, this becomes the most important part. Without a knowledge of trigonometric transformations the student cannot, for example, integrate a simple expression like $\int \sec^3 x dx$. Many other examples can be given.

Lastly, experience and opinions of engineers, engineering educators, mathematics teachers and students should not be ignored, though all of these should be supported, when possible, by data collected from actual conditions, that is, actual things which the student or engineer has to do while in training or on the job.

If data of this sort can be collected, and we hope to do some of it at least, then it should be possible to build a mathematics curriculum for engineering students which would be entirely functional. It would include such items in the field of college algebra and trigonometry that the student would need to complete the training needed for his profession and also such items needed to practice his profession successfully.

4. *The Relation of These Needs to Those of Other Sorts of Students* is important, especially in a college where it is not desirable or possible to organize two sequences of courses in mathematics. To be sure of these relations it would be necessary to also study the curricula and needs of other sorts of students. Few such studies have been made, hence what follows is merely opinion, though it is based on a considerable amount of experience.

Students other than engineering students who study much mathematics in college usually fall into two classes, those who major in mathematics and those who study it because they need it in some field like the physical sciences or statistics. It seems certain that specialists in all of these cases will need as much and probably more mathematics than the engineers, since knowledge through calculus and often through differential equations is basic for the work they must do. Hence, it should be possible to teach all groups together with perhaps a variation on the last course as is often done now in the course called *Advanced Topics in Mathematics for Engineers.*

5. *Significance of This Data in Regard to Mathematical Education.* Data of this sort, if available and reliable, should be of great value for high school teachers and administrators. Many of the items of knowledge, skills, etc. will, or should be, encountered first in secondary school mathematics. Since the mathematical maturity of a student depends to some extent on the length of time he has been familiar with and used a concept, it is of value to him to have some of these concepts introduced earlier than they now are. For example, concepts like inverse functions or trigonometric equations are often omitted from beginning courses in trigonometry or are put near the end of the course so that the student has little experience with them. Hence when he encounters them in other courses the concept is often vague or entirely unknown. If such concepts are found to be valuable for the training of engineers and scientists, they should be introduced early in the student's mathematical experience and should be encountered periodically until he is able to use them naturally and accurately. This means that they should be given due consideration by the high school curriculum maker.

It goes without saying, of course, that such items as are found necessary for an engineer's training must be introduced into the college curriculum. It could well be that the college courses as now organized are entirely adequate for such purposes. However, this will not be known until studies of the sort discussed in this paper are completed and thoroughly checked. To insure their continued functionality it will be necessary to keep the data up-to-date by periodic checking.

An Experience Program for the Training of Teachers of Mathematics at the University of Oklahoma*

By Eunice Lewis

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The Student Training Program at the University of Oklahoma

Prior to September, 1946, the training of student teachers at the University of Oklahoma consisted of a "subject centered single hour period." It was evident that the type of program which had been offered was failing to prepare our teachers for their future responsibilities. It became apparent that there was a need for an integrated experience curriculum that gave implementation of theory through varied experiences with children and adolescents in a complete school program. This could not be obtained in a "narrow and restricted one hour segment of the day."

The plan now in effect at the University of Oklahoma consists of an eight semester hour block of one-half day for one semester of integrated experiences incorporated in the following courses: two hours of Effective Teaching Procedures in Secondary (Elementary) Schools; two hours of Introduction to Student Teaching in the Secondary (Elementary) Schools; four hours of Supervised Teaching in the Secondary (Elementary) Schools.

The Effective Teaching Procedures includes general methods and orientation to teaching. Here the student teacher is given the opportunity of becoming acquainted with the program of the whole school and the interrelation of its varied activities.

The Introduction to Student Teaching is an application of the general methods to the specialized subject field. It "includes actual planning and organizing of teaching learning situations, home-school-community analysis, efficient operation and effective utilization of instructional materials and equipment, and individual and group conferences with the supervising teacher and director."[1]

The Supervised Teaching includes the directed observation and actual teaching experience of the student teacher. Here, the student teacher is given the opportunity to develop through experiences in a normal school situation under the careful guidance of a supervising teacher. It gives him an opportunity to attempt the real job without the risk of wrecking himself and the class with which he has been entrusted—at least the cost of failure, in case it occurs, can be held at a minimum.

Prerequisites for Admittance to the Student Teaching Program at Oklahoma University

Before a student may participate in the teacher training program at the University of Oklahoma, the following qualifications must be met:

1. A student must have ninety or more semester hours. This means that he will be of senior standing.
2. Ten hours of this requirement must be in the foundations courses which include: (1) The Child in American Democracy; (2) School in American Democracy; (3) Curriculum in American Democracy.
3. Recommended electives are: (1) Tests and Measurements; (2) Guidance; (3) Educational Psychology; (4) Adolescent (or child) Psychology; (5) Audio-Visual Methods.

* Read at the summer meeting of The National Council of Teachers of Mathematics at Denver, Colo., August 29—Sept. 1, 1949.

[1] Dr. Garold D. Holstine, "Professional Training for the Whole Teacher," The Nation's Schools Magazine, *The Nation's Schools Division* (the Modern Hospital Publishing Co., Inc., Chicago, Illinois, April, 1949), p. 52.

4. The applicant must be recommended by his advisor in the academic field or by his advisor in the College of Education.
5. The student must have the approval of the director and the supervising teacher of the University School.
6. He must meet certain health requirements set up by the university. This includes a tuberculin test.

What Is Student Teaching and What Are Its Objectives?

The *Handbook on Student Teaching in Mathematics* defines student teaching as, "a means of promoting growth through meeting issues, situations, and problems under wise guidance."[2]

The teacher training program and the training school may be likened to the interneship of the doctor and the hospital. It is at this point in the student teacher's training where he learns by doing; where he meets and solves difficulties through his own ingenuity and adaptability, with the guidance of an experienced teacher.

There seems to be no real need to justify the inclusion of laboratory experiences in the teacher training program. However, it is interesting to note that two major studies made within the past year are in full agreement.

At the teacher education conference held last summer at Bowling Green, Ohio, the group reporting on the Professional Preparation for Senior High School Teaching says, "the materials and practical methods in the subject-matter fields of specialization, designed to give future teachers actual experience in their practice, should be a part of every teacher education curriculum."[3] Also, in this report, it is recognized that "direct experiences in the education of prospective teachers" is important and that first hand experiences are essential.

From a report of the American Association of Teachers Colleges, we find a continued agreement in the statement, "There can be no question as to the importance of student teaching as a professional laboratory experience in the total program."[4]

Also, the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics state: "The most important element in professional training is student practice teaching carried out under the most competent supervision that can be procured."[5]

Recognizing that the future of America is dependent upon the quality of the teacher, it is most necessary that a program for the training of teachers be offered which is designed to prepare for the profession members who are competent, responsible, and well adjusted to meet the growing needs of our democracy.

Our program at the University of Oklahoma is an attempt to meet this challenge. With this in mind the following objectives were set up:

1. To provide an opportunity for the student teacher to implement and coordinate his academic and professional training.
2. To provide directed observations in which he may learn to know children and to see how they develop through the utilization of the best teaching skills and techniques.
3. To provide actual teaching experiences in a normal school situation.
4. To provide an opportunity to work in pupil-parent-teacher and in home-school-community situations.

[2] W. D. Reeve and Homer Howard, *Handbook on Student Teaching in Mathematics for Student Teachers and Their Supervisor*, The Mathematics Teacher (March, 1947, 525 West 120th, New York). p. 1.

[3] The official group reports of the Bowling Green Conference, sponsored by The National Commission on Teacher Education and Professional Standards and The National Education Association of The United States, *The Education of Teachers*, a pamphlet, 1201 Sixteenth Street, Northwest, Washington 6, D. C., 1948, p. 207.

[4] Sub-Committee of the Standards and Surveys Committee of the American Association of Teachers Colleges, *School and Community Laboratory Experiences in Teacher Education*, a pamphlet, 1948, p. 146.

[5] The Final Report of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics on *The Place of Mathematics in Secondary Education*, Fifteenth Yearbook, National Council of Teachers of Mathematics, Bureau of Publications, Teachers College, Columbia University, 1940, p. 191.

5. To provide for efficient operation and utilization of instructional materials and equipment.
6. To provide opportunity to become acquainted with the program of the whole school.
7. To provide "responsible participation" in the activities which make up the teacher's day.
8. To provide for the acquisition of knowledge of professional ethics.
9. To provide the opportunity for professional growth through the directed reading of professional literature.

The Activities Engaged in by All Trainees Consists of

I. *Junior Faculty meetings*

In order to better understand the function of the faculty meeting, its method of procedure, and its actual value, the Junior Faculty meeting is set up as a weekly occurrence. Here the students carry on the business and affairs of their own organization. Topics of a professional nature are discussed by their own members and frequently by guest speakers. Popular topics discussed are the P.T.A., its organization and function in the community, and the procedure of selecting and applying for a job suitable to one's ability. Other topics used are: modern trends in methodology and instructional materials in the specialized subject fields, school health, democratic procedures, and professional ethics.

II. *P.T.A. meetings*

In order that each student teacher will have an understanding of the correlation of teacher-pupil-parent relationship, each is expected to know the organization of the P.T.A. and to understand its function in the community. Each is required to attend at least two meetings of the P.T.A. of the University School. Frequently, many of the student teachers are asked to participate in the planning and in the execution of programs. For example, the program on health will be in charge of the physical education group, while those in music will prepare the music program.

III. *Audio-visual program*

In September, 1946, the present audio-visual program was developed under Dr. Garold Holstine, Director of the University School and Student Teacher Education at Oklahoma University.

The goal has been the effective operation and utilization of all types of audio-visual equipment. Each student thus becomes competent and confident in their use. In this training, which is required, the student teacher is taught how to select, present, and evaluate suitable films and film strips. He also learns the location of available audio-visual services in the state and the addresses from which these materials may be ordered. Each student teacher is given a series of five or more demonstrations and operational experiences with every type of equipment that he will probably find at the school where he will teach. The equipment which he learns to use includes the wire and disc recorder; the opaque projector; the 15 mm. film strip projector; the 16 mm. motion picture machine; the micro projector (glass slides); the combination record player, radio, and recorder.

IV. *School clubs and assemblies*

The student teachers are given the opportunity to participate in the organization and direction of the various school clubs. Among these are: dramatics, student council, Spanish, home economics, and mathematics. Assemblies given throughout the year are under the direction of the student teachers.

V. *Social affairs*

Since each student teacher will be expected to sponsor and direct school social affairs, he should have experience in such activities during his training period. Many school parties are organized and supervised by the student teacher, giving him practical experience for his future position.

VI. *Community activities*

One of the requirements of a teacher is to be able to take a responsible part in the activities of the community of which he is a member. Therefore, a two week unit on home-school-community relationships is offered to all trainees. Each is encouraged to participate in the Teen Town program, community drives, youth organizations, church activities, and other community activities. These contacts will help him to understand the interrelationship of home-school-community experiences and will assist in understanding and solving the common problems.

The Experiences Offered the Student Teacher in Mathematics

The specialized training offered the student teacher in mathematics consists of varied experiences.

First, there is the general orientation. During this time the student teacher becomes acquainted with the entire school staff from kindergarten through high school. He learns the use of the office forms for recording absences, tardies, and the permits of various types. He begins to understand the purposes and objectives of the school and of the Mathematics Department. This is also a time when he attempts to develop an understanding of himself, develop a wholesome outlook on life, and a deep concern for the welfare of others. In so doing, he will be more capable of understanding and working with his pupils.

The next phase of the training of the student is serving as an assistant to the teacher. Since the teacher must be aware of what constitutes a pleasant classroom environment, he is made responsible for maintaining pleasant and attractive surroundings.

He becomes acquainted with the pupils, takes roll, and keeps the attendance record. He collects needed supplies, arranging them in a convenient manner; assists the supervising teacher in either locating or constructing needed multisensory materials; keeps the bulletin board and display cases up-to-date with attractive material; takes tests with the pupils; and grades papers. He assists the supervisor during the supervised study period, frequently working with retarded pupils or those making up work.

Observation is the third step in his specialized training. The student teacher first observes his supervising teacher at work. At this time he becomes acquainted with various teaching techniques and skills used. However, according to the *Handbook on Student Teaching in Mathematics*, this "is not merely an exercise in recipe gathering."[6] He becomes aware of the whole pattern which his supervising teacher is attempting to attain through the methods utilized, and he begins to understand that each experience becomes a vital part of the whole. As situations arise he becomes conscious of an effort to set up his own solutions and to make use of his own resources and ingenuity.

After the student has worked with his own supervisor until he has become familiar with regular classroom procedure, methods, and policies in his department, he is sent out to observe other teachers at work. These observations come under the categories of the growth and development of the child from kindergarten through high school, the number concept experiences from the kindergarten through junior high school, the teaching techniques used in other subject fields, the discipline situations occurring in the different forms of classroom procedure, and off campus observations.

The growth and development of the child from the kindergarten through high school is traced by a series of observations made on each grade level. Prior to this, reading assignments have been made on

[6] W. D. Reeve and Homer Howard, *Handbook on Student Teaching in Mathematics for Student Teachers and their Supervisors*, The Mathematics Teacher, March 1947 issue, 525 West 120th, New York, p. 8.

the subject of child development and growth. The supervising teacher holds conferences with the student teacher before and after the observations. The student teacher takes special note of the behavior of the pupils of varying age levels and the manner in which the teacher handles different situations.

Tracing the number concept experiences is of most interest to the future mathematics teacher. It is quite revealing to him to see how concepts of comparison, grouping, quantity, matching etc., introduced in the kindergarten help to form the number pattern of the child. It is most helpful to the student teacher to know what type of mathematics is introduced on the various levels, how it is presented to the child, and the meanings involved.

Emphasis is placed on observing teachers in other fields. Many of these teachers use techniques and skills applicable to and effective in mathematics classes.

In order to see the influence that certain types of procedures have on the behavior of the pupils, each student teacher visits classes where characteristic methods are noted. In the English and social studies classes, discussion is predominant; in the home economics, the typing, the shops, and the art, the laboratory is used. Here the student teacher has the opportunity to observe the different forms of discipline. Does it have a positive or a negative phase? What are existing conditions which contribute toward making or preventing a discipline situation? Are the problems handled with force, rules, and threats, or through a character building program of democratic methods?

The off-campus observations consist of visits to neighboring schools of varying sizes. In this way the student teacher obtains an everyday picture of a school in action, which helps him to determine the type of teaching situation for which he is best suited.

The daily conferences which the supervising teacher holds with her group of student teachers is one of the most vital parts of the program. At this time both the supervisor and student teachers cooperatively make plans and solve problems. Each student teacher is encouraged to make contributions, thus sharing his ideas and experiences. In so doing he begins to feel that he has a part in the program. This helps to create in him a feeling of importance and security. Thus the student teacher takes one important step in his growth as a teacher.

Some of the topics discussed at these conferences are: planning procedures, observations, discipline, guidance, and child development and growth. Such discussions arise from the classes they are teaching or have observed.

This is also a period of indoctrination. The student teacher becomes aware of his great responsibility of directing the future citizens of our democratic America. He realizes that to be successful he must enjoy teaching, like mathematics, and like to teach mathematics.[7]

Much time is spent on reviews of interesting articles of a professional nature. Emphasis is placed throughout the course on the reading of professional literature for the mathematics teacher. By means of this reading he will become familiar with the achievements, contributions, and trends in teaching.

Too often, students report for their teacher training without having adequate competency in the field of mathematics; consequently, much of this conference time must be taken up in reteaching and reviewing the subject matter on the secondary level. This inadequacy is often due to an insufficient mathematical maturity, as a result of a limited program in mathematics.

Individual conferences are also necessary when the student begins his actual teaching. He must be assisted in planning his work. This is a time when he needs

[7] Howard Fehr, *Teachers of Mathematics*, THE MATHEMATICS TEACHER, Vol. XLI, No. 6, New York, p. 290.

help, encouragement, and constructive criticism. It is much wiser and kinder to work with him alone.

Since each teacher is expected to be a guidance person in his field, much conference time is given to this training. In order to better direct his pupils, he must know much about them, their interests, and abilities. He must also know the opportunities open for those interested in the field of mathematics. Consequently, he goes to the following sources for information: the Guidance pamphlet published by the National Council, Guidance literature, and the Pupil's personal data folder kept by the principal or counselor. Assistance is given in the recognition and diagnosis of the needs of the child. We do not teach our student teachers to be guidance specialists but rather make them aware of the resources in the community on which they can draw, such as the psychologist, the doctor, the speech and the hard of hearing clinics, etc.

One required problem is to make a case history of one child, in which the above resources are used. This study proves most valuable. It assists the student in knowing the background, interests, and abilities of that pupil and serves as a technique pattern for future studies of a similar nature.

It is hoped that each student teacher has completed a course in tests and measurements before entering the training program. The testing done in the mathematics classes at the University High School is an attempt to implement the theory offered in the tests and measurements course. The student teacher is encouraged to use the information he acquired concerning the construction of tests the grading of them, and the interpretation of the results.

Various forms of tests are studied and supplemented with a visit to the testing and guidance center of the University of Oklahoma. The director gives liberally of his time in pointing out characteristics of good tests, the way in which they are administered, the prices, and the manner in which this center serves the teachers over the state.

Each student teacher is expected to make his own test for the unit he teaches and to assist in making the final examination. Standardized tests are used only when it is desired to compare the achievement of the class with national norms. The construction of the test, the use of the duplicating machines, and the administration of the test are considered most essential in the student's training.

Compilation of the professional file is one of the most important experiences in our program. The information which is collected consists of valuable and useful teaching materials which are filed in a definite, organized manner. This material should prove most helpful to the beginning teacher and should serve as a nucleus around which a continuing file may develop.

The material which is expected in the file includes:

A. A brief unit outline for the entire year of all the subjects in secondary mathematics.
B. A detailed unit plan for all the units taught by the student teacher.
C. Daily lesson plans used in his teaching.
D. Objectives for each course based on the recommendations of the National Council.
E. A copy of the Grade Placement Chart from the Fifteenth Yearbook.
F. Information on the National Council of Teachers of Mathematics, including copies of the magazine and membership blanks.
G. Information concerning the National Council yearbooks, the address, price, and a list of those that have been published.
H. A paper prepared by the student on recent trends in the teaching of mathematics.
I. A list of the state adopted texts with an evaluation of one in each field.
J. Information concerning multi-sensory materials, including addresses, prices, and descriptions. This also includes lists and evaluations of films and film strips, models and various forms of teaching aids. Hand made teaching aids are included in this collection, with a description of how they are made and how they can be used.
K. A file of interesting and pertinent bulletin board material.
L. A written report of all observations made

outside of the department of mathematics.

M. A collection of interesting items concerning the history of mathematics that will enrich or assist in the presentation of the lesson.

N. All tests given by the student teachers and any sample copies of the standardized form that are available.

O. A guidance folder, including guidance pamphlet of the National Council; a reproduction oft he picture on the frontispiece of the THE MATHEMATICS TEACHER, February, 1948; guidance pamphlets from the Department of the Interior; the guidance materials on case studies; and any other material on knowing and understanding the child.

P. Reports on the readings in the professional literature.

Q. High school requirements in mathematics for graduation: those required by the state, and those required by the University of Oklahoma.

R. A card index file which is primarily a bibliography of source material, including references from professional magazines such as THE MATHEMATICS TEACHER and the yearbooks, sources of information for multi-sensory materials, and any new ideas gained in the teacher training experience.

S. Plans for a modern mathematics classroom, including a draft of the floor plan, pictures to be used, bulletin board space, lighting, and any suitable materials which would make the room more attractive and more functional.

T. Any other material which the student teacher considers useful.

The importance of planning cannot be over-emphasized. The student teacher must be led to realize that "pupil growth is, in the last analysis, dependent on the careful selection and the systematic and purposeful arrangement of activities and experiences geared to the level and abilities and interests of each pupil."[8] Consequently, the student teacher must know the pupils he is to teach before he can plan that work which will meet the needs of his class.

If careful planning is not insisted upon, the student teacher will waste much valuable time and frequently become lost and confused. This will contribute to creating discipline problems and to a feeling of insecurity prevalent among beginners.[9]

To plan effectively the student teacher must be familiar with the subject matter he is to teach, effective teaching techniques, adaptable multi-sensory materials, materials for enrichment, practical applications, and the resources of the community. He must know where to obtain the needed multi-sensory materials or how they may be constructed. If he is to be creative in his planning, he must know the subject matter he is to teach.

In making his plans he must realize that one method will not be effective in all situations and subject areas. He must learn to adapt the method to the unit and to the needs of his class.

The daily lesson plan must have that quality of elasticity which will allow it to be quickly and easily modified to meet any unexpected change which might occur. The student teacher must learn to make these transitions with a minimum of disturbance to the pupils and to himself.

The student teacher sets up the objectives and makes an outline of the year's work in each course. This is followed by the construction of the unit plan which he selects to teach. Then the unit is broken down into the daily lesson plans.

After the student teacher's plans have been approved, he is given a chance to try his wings. The supervising teacher attempts to make the general atmosphere as normal and free from tension as possible. This assists the student teacher to adjust to his new surroundings. The student teacher must be made to realize that his supervisor is a friendly guide, and that the supervisor, the class, and he are doing the job on a cooperative basis, each eager to make a success of the job at hand.

After each period of teaching, the student teacher has a conference with his supervisor, where problems which have occurred are discussed. The student

[8] Dr. Garold D. Holstine, Student teaching publications available at the University of Oklahoma.

[9] Raleigh Schorling, *Student Teaching*, McGraw-Hill Book Co., New York, 1940, p. 88.

teacher is helped in the analysis and solution of his problems.

The evaluation of the student teacher's work is a continuing process, in which growth is noted throughout his training period. The work of the student teacher is evaluated in relation to his ability and willingness to assume his responsibilities in the program.

Regularly, evaluation forms are filled out on each student, rating him on personal equipment, professional equipment, general classroom management, skill in stimulating pupil activities, and evidence of pupil growth as a result of teaching.

Another form of evaluation is a self analysis by the student teacher after he has completed his teaching. It is very essential for him to be able to make "an honest estimate of his personal and professional self."[10]

This concludes a formal list of the experiences offered in the training of our mathematics teachers although there are also many other learning situations which occur at various times throughout the program. It is felt that this should produce teachers who have a reasonable competence as beginners—teachers whose work will eventually "make a difference in the lives of others."[11]

Nevertheless, this program does not always produce the desired results, for as the sower went forth to sow his seed, "some fell upon stony places, where they had not much earth—they had no deepness of earth—and because they had no root they withered away." However, "others fell on good ground—and bare fruit—some thirty, and some sixty, and some an hundred fold."

[10] Mildred Sandison Tenner, *The Growing Teacher*, The National Education Association, Washington, D. C., p. 4.

[11] *Ibid.*, p. 14.

Why Teach Geometry?*

By Kenneth E. Brown
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Objectives as Stated by Authors

Why teach Geometry? The answer to this question has changed markedly during the last century. Mr. Shibli in his study "Recent Developments in Geometry,"[1] points out that certain authors in 1877 indicated that the principal objective for the study of demonstrative geometry was the discipline of the mental faculties and memorization of a certain body of facts. A few years later authors were beginning to emphasize the "application of facts and principles of geometry to real human need." In a previous retrospective view of the literature in the field one finds such objectives as:

> "emphasis on reasoning,"[2]
>
> "Our great aim in the tenth year is to teach the material of deductive proof and to furnish pupils with a model for all their life thinking."[3]

In fact, after an examination of such literature Dr. Fawcett says "The consensus of opinion therefore seems to be that the most important values to be derived from the study of demonstrative geometry are an acquaintance 'with the nature of proof' and a familiarity with 'postulational thinking' as a method of thought which is available not only in the field of mathematics but also in every field of thought, in the physical sciences, in the moral or social sciences, in all matters and situations where it is important for men and women to have organized bodies of doctrine to guide them and save them from floundering in the conduct of life."[4] Thus the writers in mathematics education emphasize demonstrative geometry as a way of thinking rather than an acquisition of facts.

Objectives as Indicated by the Teachers

One of the first decisions of a geometry teacher must be in regard to the purpose he must set up for the course. The experiences he provides for the pupils and final evaluation should be in reference to the objective he has determined. This point of view was stated by W. D. Reeve in these words, "The first important question for any teacher of demonstrative geometry to settle is the purpose he has in mind."[5]

Replies from a Selected Group

In 1930 Mr. Shibli made an attempt to obtain a partial answer to the question "What are the teachers' objectives for the teaching of demonstrative geometry?" He asked three hundred persons to indicate the relative importance of certain objectives listed on a questionnaire. The group selected was composed of one hundred persons who had written articles on the teaching of geometry and two hundred students working toward advanced degrees at Teachers College, Columbia University. It will be readily observed that the replies were from a selected group. From this study Mr. Shibli concludes "the aims of the class room teachers are practically the same as the aims of authors of text-

* Read at the summer meeting of the National Council of Teachers of Mathematics at Denver, Colo., August 29—Sept. 1, 1949.

[1] J. Shibli, *Recent Developments in the Teaching of Geometry*, J. Shibli, Publisher, State College, Pennsylvania, 1932.

[2] The Fifth Yearbook of The National Council of Teachers of Mathematics, *The Teaching of Geometry*, p. 86, Bureau of Publications, Teachers College, Columbia University, New York, 1940.

[3] *Ibid.*, p. 132.

[4] H. P. Fawcett, *The Nature of Proof*, p. 6, Bureau of Publications, Teachers College, Columbia University, New York, 1938.

[5] The Fifth Yearbook of The National Council of Teachers of Mathematics, *The Teaching of Geometry*, p. 13, Bureau of Publications, Teachers College, New York, 1930.

books and of other leaders in the field of secondary mathematics."[6]

Replies from Five Hundred Class Room Teachers

With permission from Mr. Shibli, I sent a questionnaire containing the objectives used in his study to seven hundred teachers selected at random from the mailing list of the National Council of Teachers of Mathematics. Perhaps it should be observed that even in this procedure there was a certain amount of selection. This was due to the fact that the teachers selected were progressive enough to be members of the only national organization of mathematics teachers devoted solely to the improvement of secondary school mathematics. It might be rightly stated that even this sample represents the opinion of the better teachers in the United States.

Table A contains the list of objectives from which the teachers were asked to select five objectives that they considered the most important in teaching demonstrative geometry to high school students. It will be seen from this table that nearly half of these teachers selected, as the

[6] J. Shibli, *Recent Developments in the Teaching of Geometry*, J. Shibli, Publisher, State College, Pennsylvania, 1932.

TABLE A

Opinions of five hundred teachers concerning the importance of certain objectives for the teaching of geometry

Objective	Rating					
	1st	2nd	3rd	4th	5th	Total
1. To develop appreciation of geometric form in nature, art, and industry.	3	2	5	9	15	25
2. To give a knowledge of the facts and principles of geometry.	21	7	4	6	9	48
3. To acquaint pupils with the contribution of geometry to civilization.	0	2	2	3	5	14
4. To develop space perception and imagination.	1	2	4	7	5	20
5. To teach the meaning of proof or demonstration.	4	4	3	3	2	16
6. To develop the habit of clear thinking and precise expression.	47	25	12	8	2	94
7. To develop the ability to analyze a complex situation into simpler parts.	3	11	12	7	6	40
8. To develop the ability to subject a statement to a severe test of its truth and validity.	1	7	11	12	4	35
9. To develop the inquiring or questioning attitude of mind.	3	3	13	9	6	41
10. To teach model proofs and develop the memory.	X	X	X	0	X	X
11. To meet college entrance requirements.	0	1	0	0	3	5
12. To prepare for the study of science and advanced mathematics.	1	7	9	10	15	42
13. To solve originals and develop the power of discovery.	1	3	6	7	4	21
14. To make clear the process of deductive thinking.	2	4	3	5	4	20
15. To train in functional thinking by means of the study of geometric relationships.	6	6	10	1	8	31
16. To make practical applications in drawing artistic designs.	X	X	X	0	0	X
17. To make applications in drawing to scale in field measurements.	X	0	0	2	0	2
18. To give pleasure and mental uplift by contact with exact truth.	0	0	1	3	2	6
19. To develop mental habits and attitudes that are needed in life situations.	7	9	4	10	7	36
20. To develop moral and spiritual ideals.	0	0	0	0	1	1

Interpretation of the table: The numbers in columns 1st to 5th indicate the per cent of the total returns that were checked in these columns. The numbers in column marked "Total" indicate the per cent of the persons that selected the objective as one of the five most important objectives for the teaching of demonstrative geometry. X indicates less than 1 per cent. Example: Objective number one was selected as first in importance by three per cent of those answering the questionnaire. Twenty-five per cent of the teachers in the survey included it in the five most important objectives for the teaching of demonstrative geometry.

most important objective, "To develop the habit of clear thinking and precise expression." In fact, ninety-four per cent of these teachers included it among the five objectives that they considered most important and ninety-two per cent rated it above the fifth in importance. The returns of this survey indicate the one outstanding objective in the teaching of geometry is objective number 6: "To develop the habit of clear thinking and precise expression."

The objective rated next in importance was objective number 2: "To give a knowledge of the facts and principles of geometry." Twenty-one per cent of the teachers selected this objective as first in importance. In general the teachers that selected objective number 2 as first in importance also included such objectives as number 7 and 12 among the next four in importance. Although one teacher in five selected this objective as first in importance it is of interest to notice that more than half of the teachers did not include it in their selection of the five most worth while objectives. Are we to conclude that half of the teachers do not emphasize the "facts and principles" of geometry in teaching the subject? It is doubtful if this is a valid conclusion.

From Table A it will be observed that the teachers are not in agreement as to most most important single objective but many teachers do agree that they would emphasize such objectives as:

- To develop the habit of clear thinking and precise expression.
- To develop the ability to analyze a complex situation into simpler parts.
- To develop mental habits and attitudes that are needed in live situations.
- To develop appreciation of geometric form in nature, art, and industry.

Since the returns from the questionnaires reported the use of thirty different textbooks, the returns were separated and tabulated according to the textbook being used. This data was compared with the data of the entire survey but no significant differences were observed. In fact, there seemed to be no relationship between the textbook used and the selection of the objectives by the teachers. Of course, in many cases the teacher does not select the textbook he uses. Many of the questionnaires contain comments concerning the inability of the teacher to select the textbook in harmony with his objectives for the course. The reasons usually stated were (1) the textbook was adopted by a state or city administration, (2) he could not find a textbook that adequately filled his needs.

Objectives as Indicated by the Pupils

After observing the objectives for the study of geometry that the teachers have stated, it might be asked "Why do the pupils think they are studying geometry?" It was impossible to go into the class rooms of the five hundred teachers who returned the questionnaire but a few class rooms were visited and the students indicated why they were studying geometry. In general the students selected as important the following objectives:

- To give knowledge of the facts and principles of geometry.
- To teach model proofs and develop the memory.
- To meet college entrance requirements.
- To prepare for the study of science and advanced mathematics.
- To acquaint pupils with the contribution of geometry to civilization.

The following objectives, which were rated high by the teachers, were seldom selected by the students:

- To develop the habit of clear thinking and precise expression.
- To develop mental habits and attitudes that are needed in life situations.

Perhaps it will be profitable if the student during the course in geometry would consider the reasons for studying geometry. He might have a better perspective of geometry if he is confronted frequently with the question "Just what are the important things I am trying to get from the study of geometry?" Why not ask the students of your classes in geometry to rate a list of objectives similar to those

found in Table A, and see if their objectives agree with yours. Whatever the results, I would be glad to receive any information you obtain in securing the opinion of your students concerning their reasons for studying geometry. If, at the end of the course, the student doesn't know the purpose of the course it is doubtful if that course has been of maximum value to him.

Are We Testing in Terms of Our Objectives

A third question which we might consider is "Are our tests in terms of our objectives?" Are we saying that the most important contribution of geometry to the child is in developing the habit of "clear thinking and precise expression" and at the same time testing his ability to reproduce a proof? Are we saying that geometry will develop mental habits and attitudes that are needed in life's situations and at the same time confining our evaluations to the properties of the triangle? From a study of informal class room tests (including my own) or "standardized tests" it would seem that they do not test for the most important objectives as selected by five hundred teachers. It almost looks as if we are giving only lip service to such objectives.

Of course, such objectives are difficult to achieve but are they impossible? If they really are our objectives, shouldn't the pupil know it? If we think they are most important, should we not try to measure our success in achieving those objectives? Pupil progress toward the objectives that the frontier thinkers in the field and five hundred teachers think important will be hard to measure. The Fifteenth Yearbook of the National Council of Teachers of Mathematics indicated the importance of trying to measure these objectives when it "indicated some activities that will serve to illustrate behaviors associated with clear thinking."[7] To measure the facts memorized and the mechanical skills acquired is a relatively easy task but to measure the effect the geometry has on the behavior of the pupil and the change in his ability to draw valid conclusions in life's situations is difficult. Our measuring sticks are crude and our results are only approximate but progress has and is being made in this direction. Dr. Fawcett has recorded his attempt at the evaluation of his objectives for teaching demonstrative geometry.[8] Many others have made similar attempts on a smaller scale that have not appeared in print. It would be helpful and encouraging if you would send to The Mathematics Teacher a report of your efforts to test for these less tangible objectives. We need to share the work we are doing. We need to share experiences which have evolved from the class room rather than those hypothetical ones created in a college office. Other teachers will welcome a chance to read a report of your experiences.

The questionnaires returned by over 500 teachers indicate that the most important objectives for the teaching of demonstrative geometry are "to develop the habit of clear thinking and precise expression" and other similar purposes. The authors of textbooks in geometry and other leaders in the field of mathematics education have expressed the same objectives. Although these objectives are considered to be the most important ones, there seems to be little indication that these objectives are evident to the pupil or emphasized in the tests. If we do believe these to be worthy objectives should not the pupils be aware of them? Should we not make every effort to see that our tests measure the students progress in terms of these objectives?

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[7] The Final Report of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics, *The Place of Mathematics in Secondary Education*, p. 22, Bureau of Publications, Teachers College, Columbia University, New York, 1940.

[8] H. P. Fawcett, *The Nature of Proof*, pp. 101–116, Bureau of Publications, Teachers College, Columbia University, New York, 1938.

Provisions for Meeting the Needs of the Poorly Prepared Student in Algebra*

By Margaret L. Berger
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(This survey was made possible by a grant from the Research Committee of the University of Alabama)

As a result of teaching college algebra to students whose high school mathematics seemed to be deficient, I became interested in the provisions other colleges and universities are making to help the poorly prepared student.

Dissatisfaction with Preparation of Students

Dissatisfaction with the preparation of students in not new with the teachers of mathematics or the teachers of any other highly organized body of knowledge. Lack of high school preparation in mathematics as the cause of student failure in college algebra seems to precede the atomic age.

In 1903, Professor James M'Clure in an address at Vanderbilt University stated that forty-four out of his class of one hundred and forty-two failed to pass the course. He gave as the reason "Insufficient preparation."[1] It should be noted that two of the deficiencies of the entering freshmen at Vanderbilt University in 1903 were "inability to complete correctly the square of a quadratic," and "lack of the knowledge of a log series" which may indicate that a higher standard of mathematical achievement was expected of the students in 1903 than is now.

In 1930, it was found that over 60% of 1700 freshmen tested could not substitute *2* for a and *3* for b in the equation $X=\frac{1}{2}ab^2$ and obtain the correct result.[2] In 1940, a study sponsored by the Texas Section of the Mathematical Association of America pointed out that even with lower standards of achievement the number of failures was increasing.[3]

The results of the tests given by the U. S. Army and Navy during the recent World War has caused the lack of mathematical mastery to stand out in bold relief.

Questionnaire to Five Hundred Institutions

To find out the provisions which the other colleges and universities are making to help the poorly prepared student, I sent questionnaires to five hundred universities, colleges and teachers' colleges. The institutions selected were those listed in the Educational Directory of Colleges and Universities by the U. S. Office of Education and whose enrollment was greater than three hundred and fifty.

The returned questionnaires indicated that most of the teachers in 1949 do not feel that the freshmen are prepared for college algebra. In fact, eighty-five per cent of the teachers answered "no" to the question: "Are the freshmen, who are required to take college algebra prepared for the course?" The teachers also indicated the per cent of the students who they thought were not prepared. Two-thirds of the returned questionnaires stated that from thirty to ninety per cent of the freshmen were not prepared for college algebra.

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* Read at the summer meeting of the National Council of Teachers of Mathematics at Denver, Colo., Aug. 29–Sept. 1, 1949.

[1] James M'Clure, "Failures in Freshman Mathematics." Paper read before the University School Conference, Vanderbilt University, May 1, 1903. *Vanderbilt University Quarterly*, Vol. III, No. 4, October, 1903.

[2] L. C. Pressey, "The Needs of Freshmen in the Field of Mathematics," *School Science and Mathematics*, pp. 238–243, March, 1930.

[3] J. M. Bledsoe, "Failure in College Freshman Mathematics." *Texas Outlook*, p. 18, October, 1940.

Placement Tests

This lack of preparation was further evidenced by the number of students who failed the placement tests. The greatest variation in answers received on the questionnaire was in response to the question: "What per cent of the students fail the placement test?" Some of the teachers indicated less than five per cent and others indicated more than ninety-five per cent of the students failed the tests. More indicated that the failure on the test is between thirty-five and fifty per cent than any other like per cent interval.

Failure in College Algebra

The opinions of the teachers concerning the preparation of the students were based not only on the results of the placement tests but also on the number of failures in college algebra and contact with the students. One-fifth of the teachers stated that they also considered the high school grades of the student in judging the ability of the students.

The per cent of students who failed college algebra ranged from one per cent to fifty per cent. More than half of the colleges (57%) indicated that fifteen per cent or less failed the course; yet a large number—one college in five—reported that more than twenty per cent of their students failed. Many of the teachers commented that in their institution when a student thought he would not pass a course he dropped the course and this accounted for the low number of failures reported from their school.

In view of the large number of failures in freshman college algebra, the critics of mathematics education might raise the following questions: Is it not very expensive to reteach such a large group of students? It it not possible to predict the success of the student in college algebra and reduce the number of failures? Is it not depressing for the student to know that twenty to thirty per cent of his class is doomed to fail? Perhaps this is one reason that many students have such a dislike for mathematics.

It is not advocated that the solution to the problem is to make the course easier by having less algebra in the college algebra course. The content of the college algebra text has been diluted and yet the failures are increasing. If we examine the popular texts in college algebra we will agree with W. D. Reeve when he said "I recently looked through a college algebra text in which well over a hundred pages were devoted to elementary algebra."[4] It is doubtful whether the answer is a condensation of high school algebra to replace college algebra.

Methods to Reduce Failures

Colleges are making attempts to reduce the number of failures in the algebra courses. Some of the teachers reported their schools were trying to partially solve the problem by using placement tests. Yet, these same schools reported as high as thirty-five per cent failures. As least they did not find the use of placement tests the entire answer.

Homogeneous ability grouping of the algebra students were considered by many teachers to be an aid in reducing failures. A study of the returned questionnaires showed that sixty per cent of the institutions reported that the students of similar ability were assigned to the same class. The criteria for this grouping, in order of frequency named were: (1) algebra placement tests, (2) amount of algebra the students had in high school, (3) the field in which the student was majoring and (4) the students high school grades. In general, the per cent of failures reported from the colleges that indicated they used homogeneous ability grouping were slightly less but even among these colleges failures of more than thirty per cent were reported.

[4] W. D. Reeve, "Coordinating High School and College Mathematics," *American Mathematics Monthly*, p. 1, Jan., 1947.

The most general procedure used by the teacher to help the student overcome his mathematical deficiencies is individual instruction from the teacher outside of the regular class period; and then if the student fails he repeats the course. The following methods that are used by the teachers are listed below in the order of frequency with which they appeared on the returned questionnaires:

1. Individual help from the teacher.
2. By taking an elementary course in algebra without credit.
3. By repeating the regular algebra course.
4. By taking a more elementary course in algebra with partial credit.
5. A laboratory period where the students receive help from the teacher or an advanced student.
6. By the help of a tutor.

Although several of the institutions reported that they were offering an elementary course in algebra with partial credit (for example, a course meeting five days a week with two hours credit), more than twice as many schools stated that they offered the elementary course but without credit. Half of the questionnaires returned indicated that an elementary course in algebra was offered in that institution either with or without credit. On several of the questionnaires on which was reported that an elementary course in algebra was not offered, there were comments similar to the following: "I might state that I have found in the past that 65 per cent of the average freshman class are unprepared to take college algebra. The common practice is to do what I am doing—to devote nearly all the time to high school topics which makes the course *college* algebra in name only." Thus, the questionnaires indicate that, in more than half the schools reporting, the student may take a course in algebra more elementary than the traditional college algebra.

Topics in Which Students Are Deficient

The teachers were asked to list the topics in high school algebra in which they considered the students especially weak. The following list of topics is in the order of frequency named: Fractions, exponents and radicals, verbal problems, factoring, understanding a formula and fundamental operations. More than forty other topics were listed but with a combined frequency less than the frequency of any one of these named above, therefore, it seems evident that the teachers consider the students *especially* weak in these topics. It should not be assumed that all the high school teachers should immediately stress these particular topics because the desirability of all these topics for high school students might be questioned. Much has been written concerning the questionable value for high school students of such topics as complicated fractions and factoring.[5] Perhaps there are other topics that should not be stressed in an elementary course in algebra for high school students. In any case, the questionnaires present the opinions of college teachers and surely the opinions of the high school teachers concerning the importance of topics in high school algebra should be considered.

It has been emphasized by many of those answering the questionnaire that the students were able to perform mechanical manipulation but lacked a knowledge of the process involved, therefore, they were unable to apply algebra to simple situations. One person commented: "I am not interested in the 'topics' in which the students are weak but rather in the lack of understanding of the basic relationships. The use of rote methods of learning algebra is to be deplored." Another commented: "The basis of difficulty, I feel, is that students are taught algebra as a series of tricks to be carefully memorized without understanding." Perhaps it would be better for the high school teacher to stress the meaning of fundamental operations and the elementary principles of algebra and leave the complicated frac-

[5] W. D. Reeve, *The Teaching of Junior High School Mathematics*, pp. 181–187, Ginn and Company, Boston, 1927.

tions and the involved processes concerning roots and radicals for a third semester of high school algebra or for college algebra.

No doubt there is other material in high school algebra that is of little value to the high school students. If this material were omitted, there would be time for a greater emphasis on the meaning of the operations performed in order to help the student translate the algebraic symbols into a meaningful language and express genuine life problems into algebraic symbols.

Certain leaders in the field of mathematics education believe that a greater mastery of mathematics could be obtained if the different branches of mathematics were correlated. Much has been written on this subject; in fact, a series of high school texts in mathematics by W. D. Reeve has just been published. The purpose of these texts are to help the students integrate the different branches of his high school mathematics.

Whether algebra is taught as a separate subject or correlated with other branches of mathematics, the task of putting ideas into algebraic language is a difficult one for the students. Algebra is a language which requires time to learn but the learning might be more effective if the student appreciate the reasoning for that language.

One of the outstanding leaders in the field of mathematics education has said "If the subject (algebra) is to be valuable, the learning should be a pleasure."[6]

It is not advocated that the course in algebra should be determined solely by the wishes of the student but it is believed that more effective learning will take place if the student sees a need for algebra. The subject will become more popular when the student is aware of the value of the subject to him. This was evidenced by the large enrollment in the mathematics classes in the army at the close of the recent war. These men were expecting to be discharged and they *elected* mathematics because they believed they would need mathematics after leaving Service. The most popular correspondence course in the Army Education Program on either the high school or college level was algebra.[7] If the student is aware of his need for mathematics the added interest may help to reduce the number of failures in the course.

The answers on the questionnaires from certain sections of the country such as California, Texas, New York, Pennsylvania, Florida and Tennessee were compared with those of the entire survey and little difference was found. Also, the results of the survey of teachers colleges was compared with the results of the entire survey but answers differed only slightly.

Summary

The returned questionnaires sent to five hundred colleges and universities indicate that there is general dissatisfaction with the mathematical preparation of the incoming freshmen. In fact, forty per cent of the teachers stated that the students' preparation was even less than it was five or six years ago.

Attempts to help the student more effectively overcome his deficiencies were: homogeneous ability grouping, individual help from the teacher, special elementary algebra courses (without credit or with partial credit), repeating the course, a laboratory period where the student received help from the teacher or an advanced student and the help of a tutor.

The interest of the teachers was evidenced by the fact that one teacher in five who answered the questionnaire, also wrote comments on the questionnaire concerning their experiences with the problem. Many persons requested the results of the survey. In spite of their interest and efforts, there seems to be a large percent-

[6] W. D. Reeve, *The Teaching of Junior High School Mathematics*, p. 170, Ginn and Company, Boston, 1927.

[7] W. E. Sewell, "Mathematics in the Army Education Program," *American Mathematics Monthly*, pp. 196–197, April, 1947.

age of failures in the algebra classes. Nearly half of the teachers reported that more than fifteen per cent of the students failed the course.

According to the opinion of the teachers returning the questionnaires, the students are weak in the following topics: fractions, exponents and radicals, verbal problems, factoring, understanding a formula and understanding the fundamental operations.

The desirability of a thorough understanding and skill in some of these topics may be of questionable value for the high school student. At least, before the high school teacher spends a great deal of time on complicated work in fractions and factoring, he should be sure that the student understands the meaning of the more simple processes. It may be that certain topics now in high school algebra should be omitted or transferred to the college algebra course. Perhaps this is a problem that could be worked out jointly by the National Council of Teachers of Mathematics and the American Mathematical Association. The Report of the Joint Commission of the two organizations was a step in this direction.[8] As Colonel Sewell has said "In any event, college professors can expect more and more of their time to be devoted to teaching what they now consider high school mathematics."[9]

[8] The Final Report of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics, *The Place of Mathematics in Secondary Education*, Bureau of Publication, Teachers College, Columbia University, New York, 1940.

[9] W. E. Sewell, "Mathematics in the Army Education Program," *American Mathematics Monthly*, p. 199, April, 1947.

Arithmetic Section

Conducted by BEN A. SUELTZ
State Teachers College, Cortland, N. Y.

IN THIS ISSUE:

Mr. Weaver of Towson, Maryland discusses the need for a broader and deeper understanding on the part of teachers of arithmetic with particular emphasis on the mathematical aspects of "understanding." How can a teacher achieve a background that truly illuminates meaningful learning? Do training programs in the teachers colleges provide this? Is there a trend away from training based largely upon a study of child development plus a limited amount of "methods" and toward a more comprehensive study of "content"? What is the ideal training program that might reasonably be achieved? Read Mr. Weaver's article and see if you agree with his major theses.

Also, this issue summarizes the discussions on arithmetic which were presented at the Regional Meeting in New York on December 29, 1949.

IN THE APRIL ISSUE:

Descriptions of several of the newer courses of study in arithmetic will be given. Also, an annotated bibliography on the teaching of arithmetic will be printed so that teachers may use it for library lists and references for summer study.

A Crucial Aspect of Meaningful Arithmetic Instruction

By J. Fred Weaver
State Teachers College, Towson, Maryland

The first[1] and especially the second[2] report of the Commission on Post-War Plans of the National Council of Teachers of Mathematics has done much to crystallize the constructive thinking and productive research of authoritative leaders in the field. Three theses from the second report epitomize the happy trend of the past few years, especially as regards arithmetic.

"Thesis 2. We must discard once and for all the conception of arithmetic as a mere tool subject.[3]

"Thesis 3. We must conceive of arithmetic as having both a mathematical aim and a social aim.[4]

"Thesis 4. We must give more emphasis and much more careful attention to the development of meanings."[5]

The background for the present discussion is established by a consideration of Thesis 4 when coupled with the following statement under Thesis 24: "It is a mistake to assume that teachers need to know only the subject matter which they will teach."[6]

Constructive leaders in the field have presented the irrefutable case for the development of arithmetic meanings. This case has been strengthened by the results of appropriate research activity. Authoritative textbook writers have attempted to provide instructional materials which contribute to the development of mathematical meanings. From this point on, the extent to which meaningful arithmetic instruction becomes a reality for children is directly proportional to the extent to which arithmetic is meaningful to the classroom teacher herself. It is an impossibility for teachers to emphasize and direct attention to the development of meanings which they themselves do not understand, or of which they are not cognizant. Furthermore, no teacher can expect her instruction to be most meaningful to pupils until her own breadth and depth of meaning transcends that which she expects to develop in her pupils.

A three-fold or three-sided problem must be faced so far as present and future teachers of arithmetic are concerned.

(1) They must recognize the necessity of meaningful instruction as a prerequisite to functional competence.

(2) They must have an understanding of meanings to be developed, both from the level of experience and maturity of the pupil being taught and from that of the teacher herself.

(3) They must be conscious of the psychological and methodological aspects of a meaningful instructional program. Although these three phases can be considered in no way mutually exclusive, it is the second to which the author wishes to direct major attention in this discussion.

When we accept the thesis that elementary teachers need to have a broader command of subject matter (arithmetic) than that which they will teach, we necessarily must include mathematical meanings in our thesis. The teacher's understandings and rationalizations must be deeper and richer than those which she expects to engender within her pupils. Assuredly, the teacher must have command of meanings to be emphasized in terms of the experiential backgrounds and maturity levels of the pupils being taught. But that is not

[1] "The First Report of the Commission on Post-War Plans," The Mathematics Teacher, May, 1944. pp. 226–232.

[2] "The Second Report of the Commission on Post-War Plans," The Mathematics Teacher, May, 1945. pp. 195–221.

[3] *Ibid.*, p. 199 f.

[4] *Ibid.*, p. 200.

[5] *Ibid.*, p. 200 f.

[6] *Ibid.*, p. 215.

sufficient. For most effective development the classroom teacher also must have an understanding of these same (and other) meanings from the standpoint of her own more mature level of thinking and rationalization.

There is common and universal agreement that throughout the elementary school the meaning of our positional number system is one which must recieve unremitting emphasis through progressive developmental stages. Starting with the simple concepts of tens and ones, pupils are led to a deepening understanding of the structure of our positional system of notation—a decimal system in which ten is the all-important "base" (or "radix"). An eventual analytical understanding of the structure of decimal numbers (e.g., 4256.73) in terms of their component parts has been found to pay big dividends for pupils.

$$
\begin{array}{rrl}
4256.73 = 4000. & = 4 \times & 1000 \\
200. & = 2 \times & 100 \\
50. & = 5 \times & 10 \\
6. & = 6 \times & 1 \\
.7 & = 7 \times & .1 \\
.03 & = 3 \times & .01 \\
\hline
4256.73 & &
\end{array}
$$

The teacher's understanding of positional notation must go much deeper than this, however. To appreciate the structure and positional nature of our decimal system with its base or radix of ten, it is to her advantage to understand the interpretation of numbers such as 4256.73 in the following manner:

$4(10)^3+2(10)^2+5(10)^1+6(10)^0+7(10)^{-1}+3(10)^{-2}$, which embodies a host of fundamental mathematical concepts and principles.

Furthermore, much is to be gained if teachers recognize the possibility of there being similar positional systems of notation which are not decimal in nature; i.e., systems in which some quantity other than ten, such as eight or twelve, may serve as the base or radix. Our sample number, 4256.73, represents four thousand two hundred fifty-six and seventy-three hundredths only in a decimal system. If the base were 8 or 12, then:

$$4256.73_{(8)} = 4(8)^3+2(8)^2+5(8)^1+6(8)^0+7(8)^{-1}+3(8)^{-2}, \quad \text{OR} \quad 4256.73_{(12)} = 4(12)^3+2(12)^2+5(12)^1+6(12)^0+7(12)^{-1}+3(12)^{-2}.$$

A positional number system is not something that is dependent upon *ten* for its general structural features. In fact the general structural principles underlying a positional system of notation are independent of the base used. Our decimal system is but one member of a large class of positional systems of notation. The true structure and power of such notation in general seldom can be appreciated fully from a study of the decimal system alone. The consideration of positional systems to bases such as 8 or 12 is of value to the teacher in developing an understanding of the *general* principles of positional notation as exemplified in our own decimal system.[7]

Yes, the teacher must have an understanding of mathematical meanings at the level at which she teaches—*and* at the richer level of her own experience and maturity.

By way of contrast to the foregoing, let us think for a moment of the Roman system of notation. From the standpoint of content to be taught, the teacher's understanding of Roman notation must embrace the ability to change Hindu-Arabic number symbols to Roman, and vice versa. But it is a sorry state if the teacher's understanding stops at this

[7] For the interested reader two things may be mentioned. First, the following expression represents the structure of any positional number in any base. The powers of the base (B) are the standard-sized groups established by that base; the coefficients (a) represent the number of groups of each standard size:

$$\cdots + a_3B^3 + a_2B^2 + a_1B^1 + a_0B^0 + a_{-1}B^{-1} + a_{-2}B^{-2} + \cdots .$$

Second, attention is called to the work of F. Emerson Andrews for an interesting, enlightening account of non-decimal positional notation. See *New Numbers* (How acceptance of a duodecimal base would simplify mathematics); Harcourt, Brace: 1935.

point, as is the case so frequently. It is to her definite advantage to realize that the *fundamental* difference between the Roman and Hindu-Arabic forms of notation does not lie in the form of written symbolism used. Rather, is the teacher intelligently conscious of the following fact with its far-reaching implications: that both Roman and Hindu-Arabic systems are truly decimal in nature, but that the former is non-positional in structure whereas the latter is positional? All too often the answer to this question must be, "No."

But let us look to another area, common fractions, for further suggestive illustration of the point at hand. Regardless of the rationalizations or explanations given to children for the multiplication and/or division of one fraction by another, certainly the teacher's understandings must be far broader and deeper. For example, a few selected concepts and principles which should be a functional part of the teacher's ability to comprehend fractional relationships at her own level of understanding include:

(1) Any fraction (a/b) may be considered as either a single quantity or number (N), or as an indicated division of one quantity or number (a) by another (b); i.e., $a \div b$.

(2) Division by any number (N) is the same as multiplication by its reciprocal $(1 \div N)$; conversely, multiplication by any number (N) is the same as division by its reciprocal $(1/N)$.

(3) A quotient is unchanged if divisor and dividend are multiplied or divided by the same quantity.

(4) The value of a fraction (a/b) is multiplied by any number (N) if either the numerator (a) is multiplied by that number (N) or if the denominator (b) is divided by that number (N).

(5) The value of a fraction (a/b) is divided by any number (N) if either the numerator (a) is divided by that number (N) or if the denominator (b) is multiplied by that number (N).

Now let us examine how these and similar principles govern, at least in part, familiar fractional operations such as the following:[8]

$$\frac{5}{6}\times\frac{3}{5} \quad \text{and} \quad \frac{7}{8}\div\frac{7}{4}\,.$$

In light of the foregoing principles each of the following relationships is valid in all respects:

$$\frac{5}{6}\times\frac{3}{5}=\boxed{\frac{5\times3}{6\times5}}=\frac{5\div5}{6\div3}=\frac{5\times3/5}{6}$$

$$=\frac{5}{6\div3/5}=\frac{1}{2}$$

$$\frac{7}{8}\div\frac{7}{4}=\frac{7\div7}{8\div4}=\boxed{\frac{7\times4}{8\times7}}=\frac{7\div7/4}{8}$$

$$=\frac{7}{8\times7/4}=\frac{1}{2}\,.$$

Or, in general rather than specific symbolism

$$\frac{a}{b}\times\frac{c}{d}=\boxed{\frac{a\times c}{b\times d}}=\frac{a\div d}{b\div c}=\frac{a\times c/d}{b}$$

$$=\frac{a}{b\div c/d}$$

$$\frac{a}{b}\div\frac{c}{d}=\frac{a\div c}{b\div d}=\boxed{\frac{a\times d}{b\times c}}=\frac{a\div c/d}{b}$$

$$=\frac{a}{b\times c/d}\,.$$

(Notice that a complete understanding of all relationships is dependent in large measure upon a recognition of the first of the foregoing five principles. Others are involved also, of course.)

Although all relationships are equally valid, in actual practice we use and teach

[8] In all multiplications in this discussion the sign "×" is to be read "multiplied by" rather than "times." The different implications of these two wordings should be obvious.

primarily (and almost exclusively) those which are "boxed." These will always "work" with maximum simplicity. Most often the other relationships, although valid, are not practical ones to use in multiplication and division examples because of unwanted complexity inherent in the terms of the given fractions. However, the lessened utility of these relationships is no adequate justification for their being excluded from the understanding of the teacher.

In terms of the second principle stated previously, it is a simple matter to give a "mature" single-step rationalization of the inversion method of division. $7/16 \div 7/8$ at once becomes $7/16 \times 8/7$, or $\frac{1}{2}$. In general, $a/b \div c/d$ becomes $a/b \times d/c$.

Again, consider how several of these principles operate in the following rationalizations for both multiplication and division. Let us assume that 5/6 is to be multiplied by 3/5. That is, 5/6 first is to be multiplied by 3 (which is done by multiplying the numerator by 3), and then divided by 5 (which is done by multiplying the denominator by 5). Thus, an application of the fourth and fifth principles forms a rationalization for the common procedure of multiplying numerator by numerator and denominator by denominator. A similar application of these same two principles can be used to account for the method by which we divide 7/8 by 7/4, for example, mostly commonly. First, 7/8 is to be divided by 7 (which is done by multiplying the denominator by 7); then, 7/8 is to be divided by $\frac{1}{4}$ or multiplied by 4 (which is done by multiplying the numerator by 4). In common practice this same effect is accomplished by inverting the divisor and proceeding as in multiplication.

As a final illustration, let us consider the fact that the inversion method of division may be viewed as an abbreviated form of a much more complete algorism. If in an example such as $7/16 \div 7/8$ we could convert the divisor to unity, our quotient would become the same as the dividend. Now 7/8 can be changed to unity if multiplied by its reciprocal, 8/7. Of course, doing this would alter the quotient. But according to our third principle, the quotient would remain unchanged if the dividend also were multiplied by 8/7. Doing such would produce a dividend of 1/2 which, when divided by 1, would give a quotient of 1/2. Thus, the following algorism is valid:

$$\begin{array}{|r|l}
\hline
7/16 \div 7/8 & = \\
(7/16 \times 8/7) & \div (7/8 \times 8/7) = \\
1/2 & \div 1 = \\
\hline
\multicolumn{1}{c}{1/2} &
\end{array}$$

$$\begin{array}{|r|l}
\hline
a/b \div c/d & = \\
(a/b \times d/c) & \div (c/d \times d/c) = \\
(a \cdot d/b \cdot c) & \div 1 = \\
\hline
\multicolumn{1}{c}{ad/bc} &
\end{array}$$

The boxed parts are those which make up the abbreviated form of the complete algorism, and which we term the inversion method.

Certainly no attempt has been made to illustrate exhaustively the major point of emphasis in this discussion. Rather, a few illustrations have been selected to typify the general nature of the broader, more mature generalizations and rationalizations which must be familiar to the elementary teacher. If children are to be truly quantitatively literate, arithmetic in the elementary school must be taught meaningfully. And if arithmetic is to be taught most meaningfully, the elementary teacher must have as a functional part of her preparation a depth and maturity of mathematical meaning as typified by the illustrative examples just cited.

Many teachers colleges and schools of education recognize the necessity for meaningful instruction in elementary mathematics as a prerequisite to quantitative literacy. Consequently, the methodology of teaching arithmetic meaningfully is becoming an integral part of the training of future elementary teachers in these professional schools. This trend is reflected

in the first list of selected references at the conclusion of this discussion. Here are listed professional books in which the methodological and psychological aspects of "meaning arithmetic" have been given primary consideration, and in which meanings and understandings are emphasized in terms of the level of maturity and experience of the children to be taught.

However, there is grave danger of overlooking an important link in the complete chain. Before students can learn how to teach arithmetic meaningfully, they must have acquired the necessary breadth, depth and maturity of background in the subject-matter of arithmetic so that it may be a functional part of their preparation. Entirely too often this background of understanding and skill is *assumed*, thus becoming an unemphasized aspect of the over-all program of training. In the vast majority of cases such an assumption of requisite background is a faulty one to say the least. All too frequently students are placed in teaching positions in elementary schools without having had any instruction in arithmetic *per se* since the days of their own elementary or junior high school study. Their point of view and degree of understanding is no broader nor deeper than that of the pupils they teach. It is true that these teachers well may have received some training in the methodological and psychological aspects of "meaning arithmetic." However, such courses generally do little to enrich, broaden and deepen the maturity of mathematical understanding of the prospective teacher. These courses rightfully are geared to a consideration of meanings and understandings from the point of view of the elementary school child, and generally not that of the college student.

But if such instruction in method is to serve its intended purpose most effectively, it must be preceded by work in elementary mathematics *per se*, in which meanings, understandings and applications are taught from the point of view of the maturity- and experience-level of the college student. Teachers colleges and schools of education definitely must include specific training of this nature in the subject-matter of "meaning arithmetic" prior to a consideration of important methodological and psychological aspects of the related teaching process.

A point of view akin to this has been forwarded clearly and positively in the statement and subsequent discussion of Thesis 25 in the Commission's second report.[9]

> "Teachers of mathematics in Grades 1–8 should have special course work relating to subject matter as well as to the teaching process . . ."

The present writer re-emphasizes strongly this point of view as expressed by the Commission. In fact, the opinion is ventured that institutions engaged in a program of teacher-training for the elementary school field must incorporate such course work into their total program much more so than they have done in the past if maximum effectiveness of over-all training is to be realized. Furthermore, such course work should be made available to many teachers-in-service as well as to all students-in-training.

Obviously some professional schools have had adequate foresight to recognize the crucial aspect of the factor discussed, and have made *appropriate* work in the subject-matter of arithmetic an integral part of their training program. However, this situation is far from a universal one. In the past, the paucity of satisfactory textbook materials giving adequate coverage to arithmetic *per se* at the collegiate level is a reflection of the status of such courses in our professional schools. However, the pendulum definitely is swinging in the desired direction. The second list of selected references at the conclusion of this discussion mentions present-day texts which reflect this necessary trend. The latter book is outstanding in its relation to the necessity for subject-matter study and preparation in which maturity of mathematical understandings inherent in arithmetic is to be achieved.

Sight is not lost of the social aim of instruction, nor of the necessity for train-

[9] *Op. cit.*, pp. 215–217.

ing in the "how" of teaching arithmetic meaningfully so that it may function intelligently in the lives of children and adults. But no teacher can make her instruction most meaningful to her pupils if she has had inadequate preparation (at her own level of maturity, experience and understanding) in the subject-matter of arithmetic. Our professional schools must face squarely the responsibility for providing *adequate and appropriate* training in this factor which is crucial to maximum effectiveness of meaningful arithmetic instruction in the elementary school.

Selected References

A. The following are typical of books which deal primarily with the psychological or methodological aspects of meaningful arithmetic instruction. Generally speaking, mathematical meanings are developed primarily from the standpoint of the pupils to be taught.

1. Brueckner, Leo J. and Grossnickle, Foster E. *How To Make Arithmetic Meaningful*, Winston: 1947.
2. Morton, Robert Lee. *Teaching Arithmetic in the Elementary School*, Silver Burdett: 1937, 1938, 1939
   Volume I—Primary Grades
   Volume II—Intermediate Grades
   Volume III—Upper Grades.
3. Spitzer, Herbert F. *The Teaching of Arithmetic*, Houghton Mifflin: 1948.
4. Wheat, Harry G. *The Psychology and Teaching of Arithmetic*, Heath: 1937.

B. The following are typical of the very limited supply of books available which present vital mathematical meanings primarily from the point of view of the teacher, not the pupil. The books are mathematical, not methodological.

1. Boyer, Lee E. *An Introduction to Mathematics for Teachers*, Holt: 1945. (A worthy book which essentially covers background for both elementary and secondary teachers. Because of its wide coverage, the attention devoted to arithmetic is much less than that which is most desirable from the standpoint of the elementary teacher.)
2. Buckingham, B. R. *Elementary Arithmetic: Its Meaning and Practice*, Ginn: 1947. (From the standpoint of the elementary teacher, this represents the most exhaustive and authoritative source of mathematical meanings presented especially for future and present teachers of "meaning arithmetic.")

Discussions on Arithmetic

National Council Regional Meeting, New York, December 29, 1949

Reported by Ben A. Sueltz
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Two interesting meetings developed the topics (1) The Contribution of Arithmetic to General Education and (2) Necessary Special Instruction in Arithmetic. The first was led by professor Van Engen of Cedar Falls, Iowa, and the second by Professor Grossnickle of Jersey City, New Jersey. Participating with Dr. Van Engen were Dr. Burch of Boston University and Mr. Bebell, a graduate student at Teachers College. Mr. McMeen of Newark assisted Dr. Grossnickle.

Arithmetic in General Education

Dr. Van Engen opened the panel with a statement describing the purposes of general education as: (1) the development of personality, (2) learning to work together, and (3) problem solving. It was pointed out that arithmetic is, in a very real sense, a means of communication among people and that lack of ideas and abilities in arithmetic hinders communication and produces personality blocks in the individual. Problem solving was broadly conceived to include being confronted with a puzzling situation, sensing the mathematics in the situation, establishing an hypothesis, assembling data, drawing a tentative conclusion, and verifying the conclusion. This is a type of problem solving that has significance and functional value far beyond the usual textbook problem.

Dr. Burch elaborated the following objectives of arithmetic in general education: (1) to be able to meet difficulties, (2) to learn to think things out for one's self, (3) to extend and build upon the natural curiosity of children, (4) to develop better work habits and craftsmanship, and (5) to learn to work in cooperation. Dr. Burch recalled how pupils are naturally curious to learn, how they can discover things for themselves and how confidence grows with understanding. He also showed how frustrations can be avoided through an arithmetic program that builds gradually and leads to higher and mature levels instead of thrusting the pupil immediately into a high level of abstraction.

Mr. Bebell raised three major issues: (1) what is the role of arithmetic in general education as compared with other school subjects?, (2) where should the emphasis in arithmetic be placed—on ideas or on skills?, and (3) what organization of the curriculum and classroom procedures should be used for learning arithmetic? He maintained that we should give more consideration to organization and teaching from the pupil's point of view and in terms of his experience than to a teacher's views of sequences. He argued that in many cases understanding of ideas and language is more important than abstract computations. We should continue to experiment with modes of reorganizing and teaching arithmetic.

In a discussion period the "lock-step" curriculum was severly criticized for its inflexibility and failure to meet the circumstance of individual indifferences found in all classes. It was pointed out that we need more materials of the "self help" type and that more attention must be given not only to finding specific difficulties of pupils but also in providing remedy and opportunity to overcome weaknesses. It was also pointed out that the spread in abilities probably was wider in ideas and concepts than it is in terms of computations and yet most of our grouping of pupils within a class is limited to grouping in terms of computational ability. Teaching, diagnosing, and remedial measures should deal

with all phases of an arithmetic program. In a discussion of the "core curriculum" it was pointed out that selection of materials must be made with due regard to the basic mathematical sequences involved. It would be unwise to teach long division before addition and multiplication even though an immediate and important situation using long division might arise. Most of the discussion tended to show the need for high competence on the part of the teacher.

Necessary Special Instruction in Arithmetic

Professor Grossnickle discussed the role of the following special materials in the learning of arithmetic: (1) real materials from school and community, (2) manipulative materials, (3) visual materials, and (4) textbook and symbolic materials. He showed how arithmetic in real situations requires the pupil to be responsible for the correctness of the outcome, to know what to do, how to do it, and to verify his conclusion. Manipulative materials such as rulers, clocks, and the abacus are useful for discovering ideas and principles. Discovery is the important thing. Visual materials such as pictures, posters, and films are valuable for children but films serve best to show the teacher what experiences the pupils should have. The class-room should serve to unify all of the features of a good program such as a cathechism, a place for drill, a laboratory, real experience from home and school and community, and visual and manipulative experiences.

Mr. McMeen raised the question as to whether or not arithmetic should be taught within or outside the social setting where it is used. He stated that beginning teachers should follow a textbook fairly closely but should supplement the text. He showed how the typical subject-area unit tends to follow textbook and sequential development. He rejected the "unit-of-experience" approach when the unit was planned solely by pupils but favored it as a type of learning when a capable teacher furnishes guidance in planning a unit. The "core curriculum" may be valuable depending upon what a teacher sees in it and what she does with it.

Discussion tended to point out the need for making arithmetic real and meaningful to children. The role of real experience and of manipulative and visual aids in giving meaning to learning was emphasized. There was disagreement as to whether or not different forms of the abacus were as useful as other devices with small children. There is a continuing need to seek ways of teaching for understanding, to obtain transfer values, and to develop a genuine functional competence on the part of pupils.

Miscellanea—Mathematical, Historical, Pedagogical

By Phillip S. Jones
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What's New About π?

In January of 1948 a new footnote, if not a new chapter, was added to the history of π.[1] At this time John W. Wrench, Jr., of Washington, D. C., and D. F. Ferguson of Manchester, England, published jointly the corrected and checked value of π computed to 808 decimal places.[2] This concludes a project begun by Dr. Ferguson in 1945 when he became interested in the correctness of the unchecked 707 decimal place value first given by the Englishman William Shanks in 1873 and revised by Shanks himself in 1874.[3]

Ferguson found errors in Shanks' value beginning with the 528th place and gave a corrected value to 620 places. He had extended this to 710 places by January of 1947.

In this latter month Dr. Wrench collaborating with Levi B. Smith published an 808 decimal place value.[4] Shortly thereafter Ferguson discovered an error beginning with the 723rd place of Wrench's value. The final 808 place value published jointly by these two computers may be regarded with considerable confidence since they did their computations independently and using different formulas. Ferguson used the formula $\pi/4=3\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{1}{20}+\tan^{-1}\frac{1}{1985}$ which he obtained from R. W. Morris but which has been shown to have appeared in 1893 in S. L. Loney's *Plane Trigonometry*. Wrench used Machin's formula $\pi/4=4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}$. This latter was also used by Shanks.

These are prodigious feats of computation and immediately raise the question of why should anyone undertake them. The famous American astronomer and mathematician Simon Newcomb once remarked "Ten decimal places are sufficient to give the circumference of the earth to the fraction of an inch, and thirty decimals would give the circumference of the whole visible universe to a quantity imperceptible with the most powerful telescope," according to Kasner and Newman.[5] The latter then give two reasons for such calculations: the hope to find a clue to the transcendental nature of π, and "the fact that π, a purely geometric ratio, could be evolved out of so many arithmetic relationships—was a never ending source of wonder." The former could not have motivated our modern workers since π was proved irrational by J. H. Lambert in 1761 and transcendental by F. Lindeman in 1882. Shanks, however, might have had some such motivation and hence it may be of interest to quote his own words from his first publication on this subject. "Toward the close of the year 1850 the Author first formed the design of rectifying the circle to upwards of 300 places of decimals. He was fully aware at that time that the accomplishment of his purpose would add little or nothing to his fame as a Mathematician, though it might as a Computer; nor would it be productive of anything in the shape of pecuniary recompense at all adequate to the labour of

[1] Prof. E. H. C. Hildebrandt originally suggested that *Miscellanea* include a note on the new value of π.

[2] "A New Approximation to π (Concluded)," *Mathematical Tables and Other Aids to Computation*, III, pp. 18–19.

[3] D. F. Ferguson, "Evaluation of π, Are Shank's Figures Correct?" Mathematical Note, 1889, *Mathematical Gazette* 30 (May, 1946), pp. 89–90.

[4] "A New Approximation to π," *Mathematical Tables and Other Aids to Computation*, II, p. 245.

[5] Edward Kasner and James Newman, *Mathematics and the Imagination* (New York: Simon and Schuster, 1940), p. 78.

such lengthy computations.—He was anxious to fill up his scanty intervals of leisure with the achievement of something original, and which, at the same time, should not subject him to either too great tension of thought or to consult books.—The Writer entertains the hope, that Mathematicians will look with indulgence on his present "Contributions" to their favorite science, and also induce their Friends and Patrons of Mathematical Studies, to accord him their generous support by purchasing copies of the work." (The book was "Printed for the author"—i.e. privately published.) Later Shanks says, "—no one, so far as we know, has hitherto been able to—and we are of the opinion that it can never be accomplished—to ascertain the *limit*, strictly speaking of the ratio under consideration."[6]

Our modern computers have not published analyses of their motives. They appear to have been actuated by intellectual curiosity and the challenge of an unchecked and long untouched computation. However, it might be noted that lengthy and rapid computations and machines to perform them are of great interest these days. For example H. S. Uhler has computed $\frac{1}{2}\log \pi$, $\log \pi$, and $\ln \pi$ to 214 and 213 decimal places for the purpose of using them later in computing tables of $\ln x!$[7] Wrench has computed tables of $\pi^{\pm n}/n$ to 206 significant figures to be used in later calculations of $\pi^n/n!$ which in turn are needed in calculating certain transcendental functions.[8] Werner F. Vogel has computed *Angular Spacing Tables*[9] for use in gearing problems which include tables giving angles in radians to ten decimal places. To compute these a many decimal place value of π was used. (He cites a 70 place value in the book.)

[6] William Shanks, *Contributions to Mathematics Comprising Chiefly the Rectification of the Circle to 607 Places of Decimals* (London: 1853), pp. v, vi, xiv.

[7] *Mathematical Tables and Other Aids to Computation*, I, p. 55.

[8] *Ibid.*, I, p. 452.

[9] Werner F. Vogel, *Angular Spacing Tables*, (Detroit: Vinco Corp., 1943, $10.00).

Notes on Older Facts

Historical discussions of π and collections of interesting formulas for its calculation are to be found in many places,[10] but two interesting items in its long history are often inadequately treated.

It is frequently stated that the Egyptians calculated the area of a circle as $(\frac{8}{9}d)^2$ which is equivalent to giving a value to π of 3.1605. Though not incorrect, such a statement by stating truths in modern notation and too concisely fails to display several interesting features of the Egyptian procedure. Actually the Egyptian in each case subtracted from the diameter of the circle one-ninth of the diameter and then squared this result. This is consistent with the Egyptian use of unit fractions; the use of $\frac{8}{9}$ is not. This second more exact statement also avoids any implication that the Egyptian had conceived of an abstract number π, a mathematical constant, in any modern sense. Further, the exact statement furnishes a plausible suggestion as to how the Egyptian arrived at his procedure. In the Rhind Papyrus the calculation of volumes of cylinders precedes the calculation of areas of circles. This fact has led A. B. Chace and others to speculate that the Egyptians may have made a circular cylindrical container and then several sizes of square prisms of the same height. They speculate that it was by comparing the liquid capacity of the cylinders and these prisms that the Egyptians determined experimentally that the prism erected on the square whose side was one-ninth less than the diameter of the cylinder most nearly approximated the volume of the cylinder.[11]

[10] Kasner and Newman, *op. cit.*, pp. 65–79. D. E. Smith, "The History and Transcendance of π," in *Monographs on Topics in Modern Mathematics* (J. W. A. Young, Ed.). (Longmans, Green, 1915), pp. 389–416.

[11] Arnold Buffum Chace, *The Rhind Mathematical Papyrus* (The Mathematical Association of America, 1927), Vol. I, pp. 35–36, 86–88, 91–92. A summary, with references, of other theories which have been advanced to explain this Egyptian procedure may be found in J. L. Coolidge, *A History of Geometrical Methods.* (Oxford: 1940), p. 11.

Another often quoted but rarely documented tale of π is that of the attempt to determine its value by legislation. House Bill No. 246, Indiana State Legislature, 1897, was written by Edwin J. Goodwin, M.D. of Solitude, Posey County. It begins as follows: "A bill for an act introducing a new mathematical truth and offered as a contribution to education to be used only by the State of Indiana free of cost by paying any royalties whatever on the same—.

> Section I. Be it enacted by the General Assembly of the State of Indiana: It has been found that a circular area is to the square on a line equal to the quadrant of the circumference, as the area of an equilateral rectangle is to the square on one side. . . ."

The bill was referred first to the House Committee on Canals and then to the Committee on Education which recommended its passage. It was passed and sent to the Senate where it was referred to the Committee on Temperance which recommended its passage. In the meantime the bill had become known and ridiculed in various newspapers. This resulted in the Senate's finally postponing indefinitely its further consideration in spite of the backing of the State Superintendent of Public Instruction who was anxious to assure his state textbooks of the use, free, of this copyrighted discovery. The detailed account of the bill together with contemporary newspaper comments makes interesting reading.[12]

[12] Donald F. Mela directed the writer's attention to the source for this data; namely, Will E. Edington, "House Bill No. 246, Indiana State Legislature, 1897" *Proceedings of the Indiana Academy of Science*, Vol. 45 (1935), pp. 206–210. Thomas F. Holgate, "Rules for Making Pi Digestible" in the Contributor's Club of *The Atlantic Monthly* for July, 1935 is also pertinent.

Have You Heard?

By EDITH WOOLSEY
Sanford Junior High School, Minneapolis, Minnesota

THE SMALL discussion groups at the summer meeting of the National Council in Denver last August brought out many interesting and valuable ideas. The topics for discussion were selected by the group leaders, and cover the entire field of the teaching of mathematics. Reports on a few will be given here with the hope that they will be of value to you, both in your own teaching and in your local mathematics club program.

1. Providing for Individual Differences in the Average Class.
   Leader: Gilbert Ulmer, Editor, *Bulletin* of the Kansas Association of Teachers of Mathematics, University of Kansas, Lawrence, Kansas.

There is quite a definite shortage of concrete material providing for individual differences. Our books are almost all "one track." Those who try to do individual teaching are handicapped by classes much too large. Ability grouping within the class is widely practiced. Some let better students help the slower ones, and use such situations to teach cooperative living and working. Others object to "student teach student" procedure and try to do all the checking, grading and individual instruction within each group. Such teachers are definitely overworked.

Many attack the problem by using differentiated assignments. The contract assignment is quite common. Some use a regular assignment and give extra credit for extra work Others use contract assignments or regular assignments and provide opportunities for enrichment A shelf of books on mathematics and related subjects can be used to good advantage as supplementary work or for pleasure.

Any good plan provides the minimum essentials for all and an opportunity for extra practice in more difficult work for better students. A plan for enrichment adds to the program.

2. Should Not Geometry Tests Include Proof?
   Leader: Ona Craft, Collinwood High School, Cleveland, Ohio.

Learning to construct logica' proofs with a series of statements and authorities is one of the most important objectives of a course in geometry. The present tendency to omit all proofs from standard tests may give both teachers and pupils the idea that proving is relatively unimportant. It is true that to make good scores on the best tests, a pupil must have learned to prove statements logically, even though no full proofs are asked for. It was agreed that teachers should emphasize, in class periods and on frequent tests of their own making, that the ability to work out a logical proof of a new geometrical statement is of the greatest importance.

3. Particular Needs of Those Preparing to Teach Arithmetic in the Elementary School.
   Leader: L. H. Whitecraft, Ball State Teachers College, Muncie, Indiana.

The personal qualities were listed as these: health, interest in and knowledge of children, cultural background, judgement and common sense, confidence in one's own ability to do successful teaching, and a sense of responsibility to the group, the school, and the community. The professional equipment should include a philosophy of teaching consistent with the times, a knowledge of what children know about numbers on entering school, a knowledge of all that teachers of grades below have taught, and the ability to check on readiness of pupils to proceed. The teacher should also have a knowledge of the subject matter of arithmetic and of how to use multi-sensory aids to teach it.

4. How Can We Provide Proper Training for Teachers of Arithmetic?
   Leader: Mrs. Lorena Holder, Public Schools, Dallas, Texas.

Everyone who attended this group meeting participated, and all agreed on the type of course that is needed. A college course planned cooperatively by the teachers in the mathematics and the education departments, and taught by a person who has had experience in elementary or high school teaching, or both, would be very valuable in the proper training of teachers of arithmetic.

5. Practice Exercises Which Present the Cumulative Aspect of Mathematics to the Student.
   Leader: Elizabeth Dice, High School, Dallas, Texas.

The leader explained her idea of the cumulative aspect of mathematics. Everything in mathematics depends upon some other kind of mathematics. In the building of this pyramid of mathematical knowledge we must show that every aspect is related in sequence. The following examples were used: the story of two, the sequence in geometry of the point, line, and plane, word problems in algebra, square root, and the tangent of an angle. Getting the students to build their own practice exercises around supporting principles causes the cumulative aspect to become a thinking side of mathematics.

6. What Should be the Content of a Mathematics Course Beyond the First Year Course of Non-college Preparatory Juniors and Seniors?
   Leader: Florence Krieger, High School, Rapid City, South Dakota

The purpose of such a course is to give the student the concept and meaning of number, to develop thinking and reasoning rather than to continue with boring drill, and to re-establish and strengthen his own confidence in his ability to work with numbers.

The group suggested the following content for this course: review of the fundamental operations with integers, decimal fractions, and common fractions; logarithms, slide rule, simple algebraic equations, formulas, simple geometric constructions, interest, areas and volumes, horsepower, square root, storey problems, and simple trigonometric ratios. However, the content of the course would depend on its length and local conditions.

7. What Devices Have You Found Successful in Creating Interest in Mathematics?
   Leader: Florence Ross, Ellsworth High School, Ellsworth, Kansas.

Students will work most diligently and effectively at tasks in which they are genuinely interested. Therefore, to create and maintain interest is one of the most important tasks for the teacher. The elements of novelty, usefulness, and of sheer intellectual curiosity are the primary stimuli for creating interest. The group discussed the following devices and their uses in creating interest: motion pictures, still pictures, filmstrips, slides, stereographs, portraits of eminent mathematicians, posters, cartoons, bulletin boards, instruments, models and other apparatus, field trips, mathematics clubs, puzzles, guessing games, trick problems, assembly programs, exhibits, contests, and scrapbooks. The mathematics room may be given an atmosphere which inspires better and happier work. While some of the values of using these devices may be measured, there are immeasurable attitudes and responses which are recognized and appreciated by both teacher and students.

8. Helping Orient the New Teacher.
   Leader: L. W. Lavengood, Director of Mathematics, Tulsa Public Schools, Tulsa, Oklahoma

Orientation of the new teachers could help to hold them in a school system. The pre-service training of teachers should be broad and thorough enough so that

the teacher will be properly placed. The in-service training could be taken care of by supervisors, conferences, extension classes, and evening classes under the supervision of experienced teachers in the system. Summer workshops could be used to plan a continuous mathematical program for grade school, junior high, and senior high. Supervision should be in the hands of a person who has the ability to work with people, who can criticize constructively, who knows both mathematics and methods, and who can create a feeling of mutual respect between teacher and supervisor. Teacher intervisiting should be a part of the school program. Many causes of teacher dissatisfaction could be eliminated by an up-to-date handbook giving full information about the school regulations and policies.

New Honor to W. D. Reeve

The accompanying memorandum was sent to one of the associate editors of THE MATHEMATICS TEACHER by Professor Broadbent who was well aware that otherwise the members of the National Council of Teachers of Mathematics would have remained in ignorance of this signal honor that has come to Professor Reeve. In publishing the item, the Council congratulates Dr. Reeve, and wishes to express to the British Mathematical Association deep appreciation of the cogency of the citation.

VERA SANFORD

THE BRITISH Mathematical Association is proud of its Honorary Members, few in number but outstanding in achievement. The recent deaths of Sir Arthur Eddington and Professor G. H. Hardy left a list consisting of Professor Borel (Paris), Professor Hadamard (Paris), Professor Loria (Genoa), and Sir Edmund Whittaker (Edinburgh).

At its Annual Meeting, held during April 1949 at Birmingham, the Association unanimously elected the following new Honorary Members:

Professor R. C. Archibald, Emeritus Professor of Mathematics, Brown University, Providence;

Professor J. E. Littlewood, F.R.S., Rouse Ball Professor of Mathematics, University of Cambridge;

Professor W. D. Reeve, Emeritus Professor of Mathematics, Teachers College, Columbia University, New York.

In proposing the new Honorary Members, Professor T. A. A. Broadbent (Royal Naval College, Greenwich) Spokesman for the Council of the Association, emphasized the high distinction which characterized their services to mathematics:—of Archibald as a great historian of mathematics, of Littlewood as a supreme master of mathematical analysis, of Reeve as the inspirer of countless teachers, the driving force behind the National Council's series of admirable *Yearbooks* and the devoted editor of THE MATHEMATICS TEACHER.

By happy fortune, H. C. Christofferson, one of Reeve's earliest pupils, was present at the meeting, and expressed his pleasure at the appreciation the Association had shown of Dr. Reeve's services to the teaching of mathematics.

Group Affiliation with the National Council

By H. W. Charlesworth
Chairman of Affiliated Groups, East High School, Denver, Colorado

Below are listed the groups that have affiiliated during the year of 1949 up to December 1, also a separate listing of groups which have renewed their affiliation. We have given the name of the group, time of original affiliation, number of members as reported in November, name and address of the president and of the secretary.

By the time this article appears there will be several other renewals and new affiliations. Newly organized state groups such as those in Minnesota, Ohio, Pennsylvania, Colorado and Arizona will very probably be affiliated by March 1 and thereby gain the privilege of sending a delegate to the Delegate Assembly in Chicago. It appears at the time of writing that will be as many as forty-five groups affiliated by March 1 and that at least thirty of these will be represented by an official delegate at the Chicago meeting.

We hope this list will be useful to all groups in getting in touch with one another. Most of these groups are well organized and are very active in the area which they serve. Many have excellent constitutions and by-laws. Some publish bulletins or newsletters. Many have expressed their willingness to give copies of their constitutions, by-laws, newsletters, or bulletins to groups that request them. It would be well for each group to correspond with the president or the secretary of several other groups and exchange ideas on matters of common concern. This kind of cooperation can be of mutual help.

If there are corrections to be made in the following listings, please let me know. Some time later another and more complete listing will be made.

New Affiliations

(completed during 1949 up to December 1)

Dallas Elementary Mathematics Association

Affiliated April 1949, 40 members
President: Mrs. Martha Jane Welch, 5315 Gatson Ave., Dallas, Texas
Secretary: Ruth Clough, 111 N. Edgefield, Dallas, Texas

Hillsboro County Mathematics Council

Affiliated May 1949, 40 members
President: H. E. Tropp, 311 San Carlos, Tampa, Florida
Secretary: Cora Lowman, 3208 San Carlos, Tampa, Florida

Mathematics & Physics Section of the Ontario Educational Association

Affiliated May 1949, 269 members
President: J. A. Sonley, Glebe Collegiate, Ottawa, Canada
Secretary: John McKnight, Scarboro Collegiate, Toronto, Canada

West Virginia Council of Mathematics Teachers

Affiliated November 1949, 100 members
President: Julia E. Adkins, 372 B. Street, Ceredo, West Virginia
Secretary: Mrs. W. O. Grimm, 1216 10th Ave., Huntington, West Virginia

Florida Council of Mathematics Teachers

Affiliated November 1949, 300 members
President: Mrs. Veda B. Attaway, Blanche Hotel, Lake City, Florida
Secretary: Mrs. Edward E. Cone, 2746 Lydia St., Jacksonville, Florida

Mathematics Section—Virginia Education Association

Affiliated November 1949, 210 members
President: Allene Archer, Thomas Jefferson High School, Richmond
Secretary: Lucille King, Thomas Jefferson High School, Richmond

Richmond, Virginia, Section of N.C.T.M.

Affiliated November 1949, 35 members
President: Mamie L. Auerbach, John Marshall High School
Secretary: Deborah A. McCarthy, John Marshall High School

Illinois Council of Teachers of Mathematics
Affiliated November 1949, 375 members
President: Helen A. Schneider, Oak School La Grange
Secretary: Mildred R. Nielsen, Cossitt School, La Grange

Southern Oregon Council of Teachers of Mathematics
Affiliated November 1949, 25 members
President: Don O. Ross, Klamath Union High School, Klamath Falls
Secretary: Eva Burkhalter, Klamath Union High School, Klamath Falls

Mathematics Section, East Tennessee Education Association
Affiliated November 1949, 60 members
President: Ed Brown, Central High School Chattanooga
Secretary: Velma Cloyd, East Tennessee State College, Johnson City

Wisconsin Mathematics Council
Affiliated November 1949, 125 members
President: J. R. Mayor, University of Wisconsin, Madison
Secretary: Effie Froelich, Steuben Junior High School, Milwaukee

RENEWAL AFFILIATIONS
(as of December 1, 1949)

California Mathematics Council
Affiliated November 1944, 275 members
President: Mrs. Beatrice Truesdale, 3026 Lomito Road, Santa Barbara
Secretary: Kay Bishop, 2628 South Jackson Ave., Garvey

Mathematics Section, Eastern Division, Colorado Education Association
Affiliated December 1934, 55 members
President: Albert W. Recht, University of Denver, Denver 10
Secretary: Rose Myrtle Humiston, Byers Junior High School, Denver

Dade County Association of Senior High School Teachers of Mathematics
Affiliated November 1945, 40 members
President: Mrs. Fleeta Gibbs, Edison Senior High School, Miami, Florida
Secretary: Mrs. Esther Heil, Edison Senior High School, Miami, Florida

Iowa Association of Mathematics Teachers
Affiliated April 1938, 100 members
President: Glenadine Gibb, Iowa State Teachers College, Cedar Falls
Secretary: Ruth Smith, Estherville

Kansas Association of Teachers of Mathematics
Affiliated April 1935, 240 members
President: Charles B. Tice, High School, Abilene
Secretary: Martha Rayhill, 2042 Massachusetts St., Lawrence

Kentucky Council of Mathematics Teachers
Affiliated February 1940, 75 members
President: Mrs. W. S. Milburn, 4523 Southern Parkway, Louisville
Secretary: Edith Wood, Route 3, Anchorage

Louisiana-Mississippi Branch of the N.C.T.M.
Affiliated June 1929, 216 members
President: J. W. McClimans, Southeastern Louisiana College, Hammond
Secretary: Annie Lester, Central High School, Jackson, Mississippi

Mathematics Section of Maryland State Teachers Association
Affiliated February 1933, 100 members
President: Margaret L. Heinzerling, Southern High School, Baltimore
Secretary: Betty Ann Gessler, Guryuus Fall Junior High School, Baltimore

Nebraska Section, National Council of Teachers of Mathematics
Affiliated January 1937, 75 members
President: Milton J. Hassel, Nebraska State Teachers College, Wayne
Secretary: Maude Holden, Ord

Association of Teachers of Mathematics in New England
Affiliated November 1940, 376 members
President: Elmer B. Mode, Boston Univ., 725 Commonwealth Ave., Boston
Secretary: Margaret Cochran, Somerville High School, Somerville

Association of Mathematics Teachers of New Jersey
Affiliated March 1940, 870 members
President: Hubert B. Risinger, Davey Jr. High School, East Orange
Secretary: Mary C. Rogers, Roosevelt Jr. High School, Westfield

Nassau County Mathematics Teachers Association
Affiliated November 1938, 109 members
President: Elaine Rapp, Oceanside Senior High School, Oceanside, N.Y.
Secretary: Alexander Sokol, Great Neck High School, Great Neck, N.Y.

Oklahoma Council of Teachers of Mathematics
Affiliated April 1931, 175 members
President: Dan Gardner, Oklahoma City Schools, Oklahoma City
Secretary: Stella Edmiston, Oklahoma City Schools, Oklahoma City

Mathemaic Section of the Texas State Teachers Association
Affiliated February 1939, 245 members
President: Ida May Bernhard, 331 W. Hopkins, San Marcos
Secretary: Lois Averitt, 504 W. Hickory St., Denton

Men's Mathematics Club—Chicago and Metropolitan Area
Affiliated June 1929, 75 members
President: David Rappaport, Lane Technical High School, Chicago
Secretary: E. L. Tierney, Tilden Technical High School, Chicago

Women's Mathematics Club of Chicago and Vicinity
Affiliated December 1929, 76 members
President: Bernice von Horn, 11434 Forest Ave., Chicago 28
Secretary: Lucile Gates, 8201 Vernon Ave., Chicago

Greater Cleveland Mathematics Club
Affiliated November 1939, 150 members
President: W. A. Nardi, East Technical High School, Cleveland
Secretary: Lena G. Robinson, Benjamin Franklin School, Cleveland

The Detroit Mathematics Club
Affiliated June 1929, 250 members
President: William C. Slemer, 13633 Montrose, Detroit
Secretary: Geraldine Dolan, 14419 Strathmor, Detroit

Wichita Mathematics Association
Affiliated October 1936, 50 members
President: K. N. Nickel, 129 South Estelle Wichita
Secretary: Laura Smith, North High School, Wichita

The Tulsa Mathematics Council
Affiliated December 1935, 48 members
President: Mrs. Muriel Lackey, Lowell School, Tulsa, Oklahoma
Secretary: Mrs. Susie Crankshaw, Roosevelt J. H. S., Tulsa

Western Pennsylvania Mathematics Teachers Association
Affiliated March 1931, 210 members
President: Miss Clementine George, 705 College Ave., Pittsburgh 32
Secretary: Miss Ethel F. Turner, 102 Scotia St., Pittsburgh 5

CONFERENCE ON THE TEACHING OF MATHEMATICS

ILLINOIS STATE NORMAL UNIVERSITY

SATURDAY, APRIL 22, 1950

Principal addresses to be given by
Professor Leo J. Brueckner, Univ. of Minn.
Professor F. L. Wren, George Peabody College

Discussion groups dealing with three levels of arithmetic, general mathematics, algebra, and geometry.

AIDS TO TEACHING

Henry W. Syer
School of Education
Boston University
Boston, Massachusetts

Donovan A. Johnson
College of Education
University of Minnesota
Minneapolis, Minnesota

BOOKLETS

B.29—The Amazing Story of Measurement

The Lufkin Rule Co.; Saginaw, Michigan
Pamphlet; 24 pages; $5\frac{1}{2}'' \times 7\frac{1}{2}''$; color; free.

Description: This unusual publication presents the history of measurement by a series of drawings in color with brief captions. The development of linear units of measure began with the crude measurements of the caveman and ends with the precision measurements of modern science and industry. The origin and length of each unit of measure such as the cubit, fathom, inch is illustrated. The relationships of these units to physical objects is discussed (for example, the inch was the width of the thumb in 1150 A.D.) as well as the relation of units to various number bases. Important events in the growth of the metric system are used to illustrate the need for accuracy and a simple decimalized system of measurement in the development of modern civilization. Part of this story involves E. T. Lufkin and the Lufkin Rule Company.

Appraisal: By using cartoons this publication simplifies and motivates the study of the development of linear measurement. It is necessarily brief but covers the common unit of linear measurement as well as many of the significant events in the history of measurement. It emphasizes that accuracy of measurement is the key to precision which is so essential today. The pictures are colorful and the captions are brief. It will be suitable for junior or senior high school with its greatest use in vocational or industrial arts classes. With the great current interest and consumption of comic magazines this pamphlet will be able to reach students' interest and reading level. The pamphlet contains five pages of advertising out of 24 which is considerable. The quality of the paper is low so that the booklet will not stand up under long usage. Although the pamphlet is primarily for industrial and vocational art student bodies above the eighth grade, it is available to all teachers in limited quantities upon request.

B.30—An Excursion in Numbers

The Duodecimal Society of America; 20 Carlton Place, Staten Island 4, N.Y.

Pamphlet; $5'' \times 8''$; 8 pages; F. Emerson Andrews, author; Free.

Description: This pamphlet is a reprint of an article appearing in the Atlantic Monthly, October, 1934. It discusses the mathematical and practical advantages of the number base 12 in popular terms. The pamphlet discusses the following topics: (1) a survey of the development of our number system; (2) the inadequacy of the number system based on ten; (3) the advantages of a number system of base twelve; (4) how to compute in the duodecimal system; and (5) the effect of the adoption of the duodecimal system.

Appraisal: This article written in an interesting style will furnish the secondary mathematics teacher with material for a worthwhile sidelight on mathematics. Besides being an excursion for the superior student, numeration systems can be used to develop increased understanding and

appreciation of our number system and fundamental processes. This pamphlet contains adequate explanation and sample problems so that the teacher as well as the student will learn the processes of duodecimal arithmetic without previous knowledge of the duodecimal system. The writer has found this pamphlet to be very usable in an eleventh grade mathematics class.

EQUIPMENT

E.25—Parts-Imparter

Exton-Aids; Box MT, Millbrook, New York

A complete set includes a teacher DOUBLE DISC, 24 student DOUBLE DISCS, two EQUIVALENCE CHARTS, and a set of instructions. The price is $2.00. Extra student DOUBLE DISCS can be purchased at $3.00 a hundred.

Description: The teacher DOUBLE DISC consists of two interlocking nine inch discs, one black and one yellow, made of good grade bristol board. The student DOUBLE DISCS are similar $4\frac{1}{2}$-inch interlocking discs. By the simple technique of cutting a radius on each disc and mutually overlapping two contrasting colored discs along these slits, a device has been constructed which generates a visual presentation of any improper fraction, any per cent up to 100, and any decimal fraction up to unity. Likewise, the concepts of angles and the measurement of angles can be visually developed. Because the discs are solid colors without markings they can be used alone only to develop a sensing of the idea of parts in relation to the whole and to provide opportunity to estimate size of parts. However, through the use of EQUIVALENCE CHART I which has 9-inch circles showing parts equal to one-half, one-third, and one-fifth, it is possible to match and get like parts on the teacher DOUBLE DISC. EQUIVALENCE CHART II has the three 9-inch circles on the top row divided into multiples of halves, thirds and fifths so that the teacher can match and dynamically produce any of the parts that are related to the "primes" of one-half, one-third and one-fifth and the pupils can then generate the same part on their student DOUBLE DISCS. The bottom row of 9-inch circles on EQUIVALENCE CHART II provides models for producing fourths, sixths, twelfths, twenty-fourths, sixtieths, hundredths and so on.

Appraisal: The interlocking DOUBLE DISCS provide a simple and neat technique for developing the ideas of parts in relation to the whole. The general ideas of the part-whole relationship can first be presented and then the concepts can be made more precise through actual matching of various sized parts with the parts that are shown on the EQUIVALENCE CHARTS. When it is possible to have comparable devices in the hands of each child, the opportunity for adequately developing ideas is increased, and this is especially true with the DOUBLE DISCS because children would constantly have to judge whether the parts they were finding were the same size as those the teacher was demonstrating. Also, because the student devices are smaller, there is excellent opportunity to emphasize that the part must be thought of in relation to the whole before an adequate concept of the actual magnitude can be achieved. Many concepts, relations and operations for fractions, decimals and per cents can be presented through using DOUBLE DISCS, so that the criterion of multiplicity of use is well met.

The devices are constructed of good materials and the contrasting colors make the parts stand out sharply. Some of the circles on the EQUIVALENCE CHARTS have many different kinds of parts represented, and the instruction manual is not complete enough to give teachers all of the help they need in recognizing what the different divisions are, so a more thorough discussion of these circles would be helpful. The EQUIVALENCE CHARTS are the key to the satisfactory use of the DOUBLE DISCS so the effectiveness of the

development with these devices is dependent on thorough instruction in the use of these charts. (Reviewed by Robert L. Burch, Boston University, Boston, Massachusetts)

E.26—Ideal Fraction Wheel

Ideal School Supply Company, Chicago 20 Illinois.

$2.00.

Description: A seven-inch solid red circle has been printed on a 9×10½ inch green card. A second green card has the seven-inch circle cut out except for a narrow vertical bisecting strip so that when the second card is placed on top of the first, the red circle is clearly divided into two halves. A third green card is like the second, except that the bisecting strip is horizontal so that when this card is placed over the previous two, the red circle is cut into fourths. A fourth green card has two diagonal strips that divide the cut-out circle into fourths, and when this is placed on top of the three previous cards, the red circle is cut into eighths. The strips on a fifth card are so arranged that when the card is placed on top of the other four, the red circle on the bottom card is divided into sixteenths. Other green cards have the strips arranged so that certain arrangements of these cards divide the red circle into thirds, sixths, ninths, twelfths, fifths, and tenths. A yellow card has half of the seven-inch circle cut out and attached so that it can pivot on the center and provide an opening of any size up to the half. This card can be placed on top of any combination of cards and through manipulating the pivoting half, the number of eighths in a fourth, the number of twelfths in a fourth and so on can be discovered. The box can be made into an easel so that the cards can be more easily viewed by the pupils.

Appraisal: This device provides an interesting approach to various fundamental ideas concerning fractions. Through use of this material, it is possible to constantly emphasize the relation of the parts to the whole and to stress the basic understanding concerning how many parts of any size make up a whole. The difficult idea of finding a part of a part (e.g., half of a half) can be neatly developed with this fraction wheel, and the discovery of equivalent fractions can be quickly accomplished. The accompanying directions do not adequately guide the teachers in the numerous possible uses of these divided circles, so that many uses of the materials will be overlooked by teachers. The easel arrangement is good. The diameter strips which are left when the circles are cut out are likely to be broken rather quickly, but the material is otherwise sufficiently durable. The pivoting half circle pulled loose from the rivet on the sample set, but this may not be typical of the general run of the devices. (Reviewed by Robert L. Burch, Boston University, Boston, Massachusetts)

FILMS

F.47—Addition is Easy

Coronet Films, Coronet Building, Chicago

16 mm. sound film; 1 reel; black and white; B&W—$45; Color—$90.00.

Content: This film shows Billy counting the money from his bank to find out whether or not he has enough money to buy paints costing 65¢ and a bat costing $1.29. The coins from the bank are arranged according to value and the addition is made on a blackboard. Coins are changed to higher or lower value coins to illustrate place value and zero as a place holder. For example, a quarter is changed to 25 pennies which in turn are used to show that 25 means two tens and five units or two dimes and five pennies. The addition is made with coins and in writing on the blackboard as the commentator describes the counting or thinking. No words are spoken by Billy, the only person appearing in the film.

Appraisal: This film emphasizes the meaning of numbers and of the process of

addition. It also shows a realistic situation in which a third grader would use addition. However, the learner should do similar activities to those performed by Billy rather than to merely observe the processes on a film. Thus, this film may be used to introduce learning activities in a third grade or to show teachers the kind of activities to use in making arithmetic meaningful. When using this film, it will be necessary to prepare the class so that they know the value of common coins, the meaning of place value and zero as a place holder. The black and white film does not show coins such as pennies and dimes as being different, otherwise the photography is satisfactory.

F.48—Subtraction is Easy

Coronet Films, Coronet Building, Chicago

16 mm. sound film; 1 reel; black and white; B & W—$45; Color—$90.00.

Content: In this film, Billy uses subtraction to find out how much money he will have left if he spends $1.94 out of $2.23 for a bat and paints. The subtraction is performed on a blackboard with the decimal points removed. The meaning of the decimal point is illustrated by changing the coins to coins of higher or lower value. The subtraction is then illustrated by taking away coins. When borrowing is necessary, the process is illustrated by changing dimes to pennies or dollars to dimes. The process of subtraction is then illustrated on the blackboard and checked by addition.

Appraisal: This film teaches the process of subtraction in a meaningful manner by showing what it means using concrete objects. The relating of borrowing to changing from one coin to another is a good illustration of place value. For example, a dime is changed to ten pennies, when borrowing from the tens place. The film may be used to introduce a third grade to subtraction with borrowing or to show teachers how to use concrete objects to illustrate computational processes. The film should not supplant the use of concrete objects in the classroom. The pupil, viewing the film will need to know coin values if he is to understand the processes described. It is difficult to distinguish the difference between coins such as pennies and dimes in the black and white print, otherwise the photography is satisfactory.

FILMSTRIPS

FS.65—Trigonometry

Jam Handy Company, 2900 East Grand Boulevard, Detroit, Michigan

35 mm. filmstrip; black and white; silent; 45 frames; $4.00.

Description: This is one of the filmstrips from the Light on Mathematics kit. As its subtitle indicates, it was intended primarily as a refresher course. There are five frames on right triangles, thirteen on the general triangle, and twenty-three on general trigonometric relationships. Sine, cosine and tangent ratios for the right triangle are pictured. Proofs are outlined for the law of sines and law of cosines for the acute triangle, for the quotient relations, the Pythagorean relations, and for the compound, double and half angle formulas. The law of tangents and the general definitions of the functions are not given. There is no indication of how the formulas are used in the solution of oblique triangles.

Appraisal: During the war the terrible urgency of the situation in education might have made this type of filmstrip useful. Now that the emergency has passed, teachers would find a more complete treatment of certain limited phases of the work more valuable. Those using the filmstrip should call attention to some inaccuracies. (Reviewed by Frances Burns, Oneida High School, Oneida, New York)

FS.66—Graph Uses

Jam Handy Organization, 2900 East Grand Boulevard, Detroit, Michigan

1943; 35 mm. filmstrip; black and white; silent; 50 frames; $4.00.

Description: The first nine frames of this filmstrip emphasize the importance of graphs to many occupations. The illustrations show a variety of uses of graphs in the business world. The next twelve frames explain how to make a simple bar graph. The graphs represent data entitled "Cities over a Million in Population" and "Weights of Various Metals." The next twenty-one frames develop graphs which show, by using two thermometers, the linear relationship between two variables. The last eight frames review principles and terminology.

Appraisal: In general the photography is satisfactory; however, some lines and numbers are difficult to read. Inclusion of more examples of the common uses of graphs in everyday life would be appropriate. This filmstrip is a useful teaching aid for introducing the study of graphs. The continuity in developing the relationship between two variables is good. The review of principles and definitions of terms at the end is helpful. (Reviewed by Ida May Bernhard, San Marcos High School, San Marcos, Texas)

INSTRUMENTS

I.18—Speed-up Geometry Blackboard Stencils

Speed-up Geometry Ruler Co., 2206 Elsinor Avenue, Baltimore 16, Maryland

$10.00 per set; $17.50 for two sets.

Description: Three tempered masonite templets and a combination L-square and protractor comprise the blackboard drawing set. The first stencil contains cutouts for drawing an isosceles triangle, a right scalene triangle, and a regular pentagon; the second is used for reproducing an irregular hexagon, obtuse triangle, and equilateral triangle; the third, for drawing a right isosceles and acute scalene triangle and regular hexagon. Each of the figures is lettered. A twenty-four-inch ruler is printed upon the top edge of each of these three plates. The first plate contains also instructions for using the set.

A masonite L-square has a protractor cleverly superimposed upon it. Markings upon the protractor and the other plates are printed in heavy black ink which offers poor contrast against the dark masonite. The numbers and letters are hardly legible even when viewed closely.

Each of the figures drawn with the stencil is approximately seven inches in height—large enough to be seen from all parts of an ordinary classroom. Holes are provided in the stencils so that they may be hung in a convenient place. They are attractive and give the room a mathematical appearance.

Appraisal: The set may be used for blackboard drawings or for reproducing diagrams on large charts for class display in introducing polygons. Their good appearance may also add to the atmosphere of the mathematics room although such use would hardly be important enough to justify purchasing the set. The L-square and protractor seems to be the only necessary instrument in the set, for with it, all the other figures may be drawn. There is little need of purchasing four templets for drawing six triangles, a pentagon and two hexagons. Ten dollars is an exorbitant price for the set of templets which may not be purchased separately. (Reviewed by Bernard Singer, Hyannis, Massachusetts)

MODELS

M.11—Slated Globe (No. 210)

J. L. Hammett Company, Kendall Square, Cambridge 42, Massachusetts

8 inch diameter; $12.00.

Description: The hand made and hand mounted globe is made of a composition material covered with blackboard slating which neither cracks, peels, nor becomes glossy with age. The globe has horizontal and vertical wooden braces within to maintain the spherical shape. The sphere is mounted upon a simple table stand and inclined at a twenty-three and one-half degree angle. It is unobstructed by the mounting. The blackboard itself is of good

quality. Its size is objectionably small, others are available with twelve, sixteen, and eighteen-inch diameters. The larger globes are more expensive. Floor stands or hanging mountings may be obtained for any of the globes.

Appraisal: The spherical blackboard is a necessary piece of equipment in secondary schools whose curricula include introductions to spherical geometry and trigonometry. It is likewise a convenient device for explaining simple navigational and astronomical concepts, such as the distance a point on the surface of the earth moves due to rotation of the earth, the angular concepts underlying circles of latitude and longitude, reasons for establishing time zones, etc. A slated globe is quite useful in any mathematics department. The model here described will satisfy most secondary school needs. It is relatively inexpensive and should last indefinitely if not abused. (Reviewed by Bernard Singer, Hyannis, Massachusetts.)

Correction of Review of F.39—Maps We Live By (October 1949)

An error in this review has been called to our attention by Mr. J. N. MacInnes, Assistant Headmaster, St. Andrews School, Middletown, Delaware. In regard to the statement in our review ". . . and maps of the ocean's floor made with the assistance of radar . . . ," Mr. MacInnes makes this comment: "Radar will not penetrate water. Radar as used today will detect targets on the surface of the earth or in the air above, but will not show anything below the surface of the ocean or any other water, unless that object in some way breaks through the water surface. The authors of the article in question are confusing Radar with another electronic type war-time development called 'Sonar,' which makes use of sound and not electronic type transmissions."

We are glad to receive comments from our readers about our reviews in "Aids to Teaching." In this way we learn the interests and needs of fellow teachers of mathematics as well as obtain suggestions for the improvement of our reviews. We regret that an incorrect statement appeared in the review "Maps We Live By" by Norval Norse. Although the reviews written by our contributors represent their personal views, we have confidence in their material since we accept reviews only from qualified people. However, this does not excuse our failure to catch the error regarding the use of radar in mapping.

The Tenth Summer Meeting of

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

THE UNIVERSITY OF WISCONSIN
MADISON, WISCONSIN

AUGUST 21–4, 1950

The Tenth Summer Meeting of The National Council of Teachers of Mathematics will be held at The University of Wisconsin, Madison, Wisconsin, August 21–4, 1950. Meetings will be held on the campus of the University and rooms will be available in the University dormitories. A complete program will appear in the May issue of THE MATHEMATICS TEACHER. For further information please write to J. R. Mayor, North Hall, The University of Wisconsin, Madison 6.

To the Members of the National Council of Teachers of Mathematics:

It is with deep regret that I announce the retirement of Professor W. D. Reeve as editor and business manager of The Mathematics Teacher. Those of us who attended the 1949 Annual Meeting of the National Council at Baltimore recall how sorry we were to learn that Professor Reeve had been ordered to the hospital for observation and treatments in March, and we all wished him a speedy recovery. He promised to be one of our speakers at our summer meeting in Denver, and he looked forward to spending a part of the summer sightseeing in the Rocky Mountains. However, when he found early in July that he could not make this trip, he wrote us immediately and suggested that we secure someone else to speak in his place.

Despite the fact that he was confined to the hospital, he continued to carry on the work of editing the Teacher. But late in November, he wrote that he felt that the time had come for him to ask to be released from his contract at the close of this school year. Early in January, it was agreed further that his daughter, Mrs. Katherine Reeve Girard, be appointed acting editor and business manager until the end of June, 1950.

It is necessary for the Board of Directors to determine the future publication policy of the Council and to appoint a new editor and an editorial committee. To aid the Board in this work, a special committee has been appointed of which Miss Mary Potter of the Racine Public Schools, Racine, Wisconsin, is the chairman. Additional members of the Committee are: Dr. H. C. Christofferson, Miami University, Oxford, Ohio; Miss Veryl Schult, Wilson Teachers College, Washington, D. C.; Dr. Carl Shuster, State Teachers College, Trenton, New Jersey; Mrs. Marie Wilcox, Washington High School, Indianapolis, Indiana; and Dr. F. Lynwood Wren, Peabody College for Teachers, Nashville, Tennessee.

Members of the National Council of Teachers of Mathematics are invited to send their suggestions relating to our publication needs directly to the chairman or to any member of the Committee. Many letters have been received already, and additional correspondence is welcomed.

On behalf of the National Council, I want to take this opportunity to express our sincere appreciation for the care and devotion with which Professor Reeve has served as editor of The Mathematics Teacher since 1926. During that time, he has not hesitated to do everything possible to provide a real service to mathematics teaching in each of the monthly issues of the publication he served so well.

E. H. C. Hildebrandt
212 Lunt Bldg.
Northwestern University
Evanston, Ill.

Anniversary Publication

A Half Century of Teaching Science and Mathematics is being published by the Central Association of Science and Mathematics Teachers to commemorate its fiftieth anniversary.

The book traces the development of science and mathematics teaching during the past fifty years. Walter Carnahan is editor-in-chief of the books. Authors include Edwin W. Schreiber, Glen W. Warner, E. R. Breslich, Ira C. Davis, Allen F. Meyer, Jerome Isenbarger, John C. Mayfield, G. P. Cahoon, and J. S. Richardson.

The book will sell for $3.00 but a discount is offered for prepublication orders. Orders received before September 1, 1950 will be billed at $2.50. Members of the National Council of Teachers of Mathematics are invited to take advantage of this offer by sending their orders to Ray C. Soliday, Box 408, Oak Park, Illinois.

IN OTHER PERIODICALS

By Nathan Lazar
Midwood High School, Brooklyn 10, New York

Arithmetic 1949

Supplementary Educational Monographs, No. 70, November 1949. The University of Chicago Press.

1. Morton, Robert Lee, "The Place of Arithmetic in Various Types of Elementary School Curriculums," pp. 1–20.
2. Mott, Sina M., "Letting Arithmetic Function in the Primary Grades," pp. 21–25.
3. Koenker, Robert H., "Arithmetic Readiness for the Primary Grades," pp. 26–34.
4. Wilburn, D. Banks, "Methods of Self Instruction for Learning Arithmetic," pp. 35–43.
5. Wilcott, Gladys M., "Classroom Experiences with Pupil Participation in Teaching Arithmetic," pp. 44–54.
6. Buswell, G. T., "Methods of Studying Pupils' Thinking in Arithmetic," pp. 55–63.
7. Glennon, Vincent J., "Testing Meanings in Arithmetic," pp. 64–74.
8. Rogers, Don C., "Co-operative In-service Studies in Arithmetic," pp. 75–79.
9. Hartung, Maurice L., "Major Instructional Problems in Arithmetic in the Middle Grades," pp. 80–86.
10. Lazar, Nathan, "A Device for Teaching Concepts and Operations Relating to Integers and Fractions," pp. 87–100.

Miscellaneous

1. Adamas, B. R., "How We Play Store: Primary Number Experiences," *Instructor*, 58: 11, October 1949.
2. Buell, C. E., "Make Related Mathematics Related," *Industrial Arts and Vocational Education*, 38: 319, October 1949.
3. Coleman, A. J., "Faith and Mathematics," *Christian Education*, 32: 126–130, June 1949.
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5. DeMay, A. J., "Fractions in the Primary Grades," *Catholic School Journal*, 49: 27–28, Jan. 1949; 49: 98, March 1949; 49: 169–170, May 1949.
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7. Holmes, D., "Experiment in Learning Number Systems," *Educational Research Bulletin*, 28: 100–104+, April 1949.
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11. Klotz, C. E., "Mathematics Clubs for High Schools," *School Activities*, 21: 59–61, October 1949.
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13. LeBaron, W. A., "Study of Teachers' Opinions in Methods of Teaching Arithmetic in the Elementary School," *Journal of Educational Research*, 43: 1–9, September 1949.
14. Levin, D., "Consistency in Teaching Mathematics," *New Era*, 30: 117–119, June 1949.
15. Mary Adelbert, Sister, "Meaningful Arithmetic in the Primary Grades," *Proceedings of the National Catholic Education Association*, 1949: 423–425.
16. Mary Constantia, Sister, "Al-geo-trig, Mathematics Paper," *Catholic School Journal*, 49: 18, January 1949.
17. Mary Joan of Arc, Sister, "Decimal Rummy: Teaching Parts of a Dollar," *Catholic School Journal*, 49: 203, June 1949.
18. Mary of Lourdes, Sister, "Mathematics and the Physical Sciences," *Catholic School Journal*, 49: 58–59, February 1949.
19. Mary Mark, Sister, "Number Readiness in the Primary Grades," *Proceedings of the National Catholic Education Association*, 1949: 417–419.
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26. Olander, H. T., "Predicting Arithmetic Achievement," *Journal of Educational Research*, 43: 66–73, September 1949.
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